71-5479

RILEY, Robert T., 1937-
A CASE STUDY OF A DIFFERENTIATED OLIGOPOLY:
THE COMPUTER TIME-SHARING INDUSTRY.

University of Cincinnati, Ph.D., 1970
Economics, general

University Microfilms, Inc., Ann Arbor, Michigan

A CASE STUDY OF A DIFFERENTIATED OLIGOPOLY: THE COMPUTER TIME-SHARING INDUSTRY

A dissertation submitted to the

Division of Graduate Studies
of the University of Cincinnati

in partial fulfillment of the
requirements for the degree of

DOCTOR OF PHILOSOPHY

1970

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August 14 19 70

I hereby recommend that the thesis prepared under my supervision by Robert T. Riley

entitled A Case Study of a Differentiated Oligopoly: The Computer Time-Sharing Industry

be accepted as fulfilling this part of the requirements for the degree of Doctor of Philosophy in economics

Approved by:

Form 668—Grad. School—1M—3-69

PREFACE

The origin of this study can be traced to the summer of 1968 when the Quantitative Analysis department at the University decided to purchase some time-shared computer services. This department was planning to offer a series of executive level seminars designed to demonstrate modern techniques of quantitative management as implemented by computer time-sharing. Several alternative services were investigated, and immediately the problem arose of interpreting the nominal prices which were quoted. Some informal testing seemed to indicate that the user costs might be quite different from system to system. Since in this situation there was a binding time constraint, a limited purpose model was used, and the apparently best service was chosen for the specialized job at hand. Left unanswered though were a number of broader economic questions.

These apparent cost discrepancies (for example, 200 to 400 per cent) were quite striking to me; my economic training immediately caused me to wonder how buyers could allow such discrepancies to exist in the market place if, in fact, these discrepencies were typical. Since I was fortunate enough to have accumulated from many sources the computer training necessary to study this rather technical market, I decided, with the blessing of my dissertation committee, to pursue this topic. The following study is the result of that decision. I hope to repeat this study at one or two year intervals to accumulate the data necessary to conduct dynamic analyses of the effective prices in this industry.

I would like to acknowledge my substantial debt to Professors Gordon Skinner, Albert Simone, and Joseph Gallo. All of these gentlemen contributed freely of their time and deserve considerable credit for whatever merits this study may have. Obviously, the fault for any shortcomings must rest with me.

I would also like to thank my many friends and acquaintances in the time-sharing industry who spent many hours of time with me in discussions. Without their advice and access to their systems, this study could never have been made. These people are too numerous to name; however, I must single out Mr. Thomas Moore for his especial help.

Finally, I thank my wife Connie for both aid and indulgence far beyond the call of duty. She deserves the ultimate credit for making this study possible.

R.T.R.

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The ratio of hypothesis to reasonably persuasive confirmation is distressingly high in all economic literature, and it must be my chief if meager defense that I am not the worst sinner in the congregation.

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CHAPTER I

COMPUTER TIME-SHARING

1.1 Introduction

The main purpose of this first chapter is to define the goals of the study along with the methodology used to pursue them. However, since the computer time-sharing industry is rather young, many readers are probably not too familiar with some basic concepts which will be helpful in understanding various parts of the analysis. Therefore, we will review these concepts before moving on to the primary topics of the chapter.

1.2 The Nature of Computer Time-Sharing

Computer time-sharing normally refers to the simultaneous usage of a computer by a number of people--40, 60, 100, or more. On a commercial (i.e., rental) system, the user first dials the system's telephone number; hopefully, the computer "answers." The user then connects his terminal, often a Teletype machine,[1] to the telephone and proceeds to communicate with the computer. When he is fin-

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[1]It was estimated in early 1969 that 95 per cent of the communications terminals being used in the country were produced by the Teletype Corp., a Western Electric subsidiary. See "GE Steps on Teletype's Toes," Business Week, April 5, 1969, p. 52.

ished, he signs off the computer system and hangs up the phone.

Time-sharing is technically possible as long as the computer involved is fast relative to the average load placed upon it by the users. This user load is somewhat controlled by the user being restricted to a low speed terminal;[1] this restriction effectively controls his maximum access rate to the system. For example, if a computer has 40 simultaneous users, it must be able to meet their needs by devoting, on average, 2-1/2 per cent of its time to each. The words, "on average," are very important here. Over any one minute period, 10 users might be sitting at their terminals pondering the computer's last response, 10 more might be receiving output, and another 10 might be entering input. These 30 users are taxing the computer's resources rather lightly, leaving most of its computing power to serve the remaining 10 users. Also, at many times, the actual number of users might be substantially less than the maximum the system can handle; this could, of course, ease the burden on the machine even more. On the other hand, users can

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[1]Low speed remote terminals are normally 10 to 15 character per second machines (the standard Teletype is 10). A 10 character per second rate is the equivalent of 100 words per minute on output; the speed on input is a function of the typing skill of the operator. The industry is currently showing signs of accommodating faster terminals; the next standard step will be 30 characters per second which is 300 words per minute.

overburden a machine and reduce the quality of its performance substantially if they all impose heavy computational burdens at once.[1]

Time-sharing is attractive to users for many reasons. To better analyze these, let us look at the facilities which the typical commercial time-sharing service provides. Usually, the user is able to do all of the following:

1. enter programs vi . .s terminal in one or more fairly standard computer languages,
2. discover and correct syntactical and logical errors in his program,
3. correct these and other errors from his terminal either by editing or replacing faulty statements,
4. enter his input data from the terminal and receive his program results at the terminal,
5. supply information when it is requested by a program which is executing,
6. save his program on s. ne auxiliary storage device,
7. ·tilize this same ... liary storage for data files,

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[1]A good summary of how computer time-sharing works is presented by James Ziegler in Time-Sharing Data Processing Systems (Englewood Cliffs, N.J.: Prectice-Hall, 1967).

8. and use the system's library programs, if suitable, in lieu of writing his own.

This is a rather basic list; it does not begin to detail all the capabilities which various services offer. Instead, it represents capabilities which any service must offer to even be considered a time-sharing system. To understand the attractiveness of these capabilities to the user, we will review the evolution of computer time-sharing.

1.3 The Evolution of Computer Time-Sharing

The electronic digital computer is actually a rather recent invention. Although there were conceptual pioneers, such as Babbage, whose foresight extended beyond the technologies of their times, almost all the developments in electronic digital computers have taken place since the mid-1940's.

In reviewing the evolution of computer time-sharing, we will focus upon the changing relationship of man to the machine over the past 25 years. If computers are to effectively serve man (rather than the reverse), man must be able to communicate his instructions and information to the computer in a manner fairly convenient to him; however, it is axiomatic in the computer area that increases in the convenience to man incur definite economic costs. Yet as technology shifts, so do these costs, thereby altering the optimum man-machine relationship (or interface as it is often called). There has been almost a 360 degree swing in this interface from the earliest days of computers to the present. The three major steps roughly correspond to the "generation" terminology used by those in the computer field, although these generational distinctions are normally based upon the evolving electronics stages of computers--vacuum tubes to transistors to microcircuits.

The first generation computers, characterized by vacuum tube electronics, executed instructions with speeds in the milli-second (thousandth of a second) range. When the earliest of these machines were built, the only people capable of using them were their designers and builders. Unlike today, when programs (series of instructions) are commonly fed to the computer in convenient forms such as punched cards or magnetic tape, programs then consisted of temporary circuits, hand wired on a board; when the program was to be changed, the wiring on the board was altered. In these early stages, the relationship between the user and the machine was about as close as could be possible. If one is altering the wiring of the computer, this is about the extreme in "hands-on" operation. The fantastic growth and technological development in this industry has been economically possible only because of developments which allowed more users with progressively lower levels of skill to effectively harness computer power.

A major advance occurred when programs could be entered using the same input medium as for data (e.g., cards or paper tape). At this point programs were being written in "machine language." This means that the program was entirely in numeric form (sometimes binary) and immediately acceptable to the computer which was capable of accepting only numeric information. Programs were still very difficult to write; consequently, potential users were still few. The normal procedure was for the user to write his program, convert to the input medium, and then try it. Since a new and nontrivial program almost never works on the first attempt, the user would then sit at the machine console and attempt to determine where the program had gone wrong. Still, the relationship between man and machine was a close one; the response to his alterations in his program was immediate. It is probably the most efficient use of man time but certainly a highly inefficient use of machine time. When a computer is used in this way, it spends far too much time idly awaiting response or decision from the man at the operating console.

As the second generation machines were developed, computer speeds increased greatly. Also, the development of somewhat simpler languages increased the number of users. It became obvious to those responsible for supervising and optimizing computer usage that the previous inefficient methods of usage could not economically continue.[1] A technique called batch processing was developed which involved a supervisory program being placed in control of the machine at all times. Under the control of this supervisory program, the users' programs, commonly called problem programs, were run in a sequential fashion. If there was a severe error in one of the problem programs, the supervisory program would issue a message, flush the remaining input, if any, from the system, and continue to the next problem program in line.

This system operates best in a closed shop environment in which: (1) the machine is run by professionals, and (2) the problem programmer usually does not even see the machine. Under this system, the programmer writes his program, prepares the input medium, and then submits it, not to the machine, but to a clerk. At some later time, he receives the output of his attempt. The loss of man-machine interaction is obvious. Depending upon the "turn-around" time, which may vary from minutes to days, the thought processes can be substantially different from the "hands-on" situation. The utilization of the computing power of the

[1]Much of the development during these days was taking place in university environments where the economic costs of hours wasted by faculty or students carry little weight relative to dollar expenditures.

computer is substantially increased, and the total computer time necessary to develop and run a simple program is undoubtedly decreased; however, the elapsed time from the submission of the user's first attempt to the eventual return of correct output is usually increased quite substantially.

During the latter part of the second generation, easier-to-use computer languages were developed; these simpler[1] languages attracted many more new computer users. One characteristic of these new users was that, both in training and subsequent stages, they often provided large numbers of shorter programs with high input/output (I/O) requirements. Many of those people who had very long and laborious computational problems had already become computer users despite all the difficulties. To service the increasing number of new users with high I/O requirements, most third generation systems allow the computer to have in its memory the programs of a number of users. At any one instant, the computer still is limited to working on the program of a single user; however, when a user is in need of an I/O operation, the computer starts the I/O operation and immediately switches its computing capabili-

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[1]These "simpler" languages (e.g., FORTRAN) are quite sophisticated and complex in terms of their internal workings; however, for the user, they are reasonably simple to use.

ties to another user. To appreciate the need for this elaborate scheme, one must realize that the third generation computer is performing its operations in microseconds (millionths of a second). Obviously, all this switching must be done under a complex supervisory program, and the time required for this program to perform its functions represents an overhead burden on the system. In modern computer installations, the cost of this burden is outweighed by the savings from the improved machine utilization which is obtained. The name commonly given to the technique is multiprogramming or multiprocessing. It is the staple method of batch processing for modern medium-to-large scale computer installations.

Let us stop for a moment and review the trends we have discussed:

1. The simplification of computer languages has attracted many new users.
2. The utilization of the computer has improved in a hardware sense.
3. The interface between man and machine has deteriorated with substantial turn around times being the rule.

These trends had become obvious by the early 1960's and they set the stage for the development and acceptance of computer time-sharing.

Computer time-sharing allows the close man-machine relationship of the first computer generation. A single large machine is able to give to numerous users a level of service nearly as good as they could obtain if they were the only user. This close relationship is vital to the user if either or both of two needs are important:

1. the need for fast turnaround,
2. or the need for interaction with the program being executed.

Interaction refers to the ability of the user to examine his output and to communicate his decisions as to further processing or data changes based upon that output. For instance, an economist using time-sharing to implement an econometric model might wish to try various values for the investment multiplier, not knowing what new values to try until previous results are known. There is an implicit time limit contained in this concept of interaction. The user must be able to receive his results and return his decisions with delays of only seconds or minutes for a system to be truly interactive.[1]

We see then that technological developments have allowed the concept of computer time-sharing to become a reality. The impetus for development arose from the desire

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[1]Even a batch system becomes "interactive" if we allow the delays to be in terms of hours or days.

to improve man's ability to communicate with the modern computer.

1.4 The Economic Basis of Computer Time-Sharing

Even though time-sharing may be both technologically possible and conceptually attractive, these factors alone do not make it economically feasible. As we will discuss later, there are certainly a number of reasonable substitutes for time-sharing, some of which have fairly high cross-price elasticities of demand relative to time-sharing; therefore, time-sharing sellers must be able to produce their services at a marketable price level, taking substitutes into consideration. The discussion of the economics of sharing computers could fill a volume by itself, especially when the Pandora's box of communications costs is opened; we will limit this discussion to the basic analysis since the refinements are not essential to this study.[1] We are focusing here on the factors which allow shared computer time to be marketed at a reasonable cost.

The dominant economic factor is usually referred to

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[1]Readers interested in pursuing this topic might find the following helpful: Bernhard Schwab, "The Economics of Sharing Computers," Harvard Business Review, Sept.-Oct., 1968, pp. 61-70; William F. Sharpe, The Economics of Computers (New York: Columbia University Press, 1969), chap. 9; Walter F. Bauer and Richard H. Hill, "Economics of Time-Shared Computing Systems," DATAMATION, November, 1967, pp. 48-55, and December, 1967, pp. 41-49.

as the "square law" or "Grosch's law."[1] This "law" gives a rough estimate of the economies of scale in computer hardware. The law in simplified form states that, for a given generation of hardware of a particular manufacturer, differences in speed vary with the square of differences in cost or:

$$E = \left(\frac{1}{K^2} \right) C^2$$

Where: C denotes the cost of a computer system,

E denotes the effectiveness of the system,

and

K denotes some constant.

For example, a machine that costs twice as much should be roughly four times as fast.[2] This law states a hyperbolic relationship, not the parabolic one necessary for a typical "U" shaped long run average cost curve. Therefore, the law is only accurate over the downward sloping portion of the average cost curve; however, for an obvious economic reason, this covers almost all computers built. Rather than buying

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[1]Sharpe, Economics of Computers, p. 315.

[2]In "Economics of Sharing Computers," p. 62, Schwab assumes a "cube" law which produces an even more accentuated relationship. Sharpe, however, in Economics of Computers, pp. 315-22, cites several studies of third generation machines which tend to validate the square law for balanced blends of computer tasks.

a computer on the upward sloping portion, the buyer can achieve lower costs by simply adding computer power of the type found near or at the bottom of the downward sloping portion. Only the quite large machines or super-computers lie beyond the minimum point on the actual average cost curve.[1] The costs of building and supporting these super-computers are extremely high for the following reasons:

1. The demand is normally small--often less than 10 machines--which gives only a small base over which to spread high developmental costs.
2. These machines are normally custom-tailored to the buyer's needs even though they might nominally bear the same model number.
3. The size and speed requirements usually dictate that the latest and most expensive technology must be used.

The super-computer is economically acceptable to the buyer only if he has very specialized requirements (e.g., the need to solve huge problems) which can only be met by a super-computer.

This square law can be viewed as an approximation of the long run average cost curve facing a computer user; the short run cost curves would be typical parabolic functions each representing the usage of a different particular

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[1]Sharpe, Economics of Computers, pp. 316-20.

computer. The exact curvatures of the short run cost curves could vary considerably depending upon various factors such as:

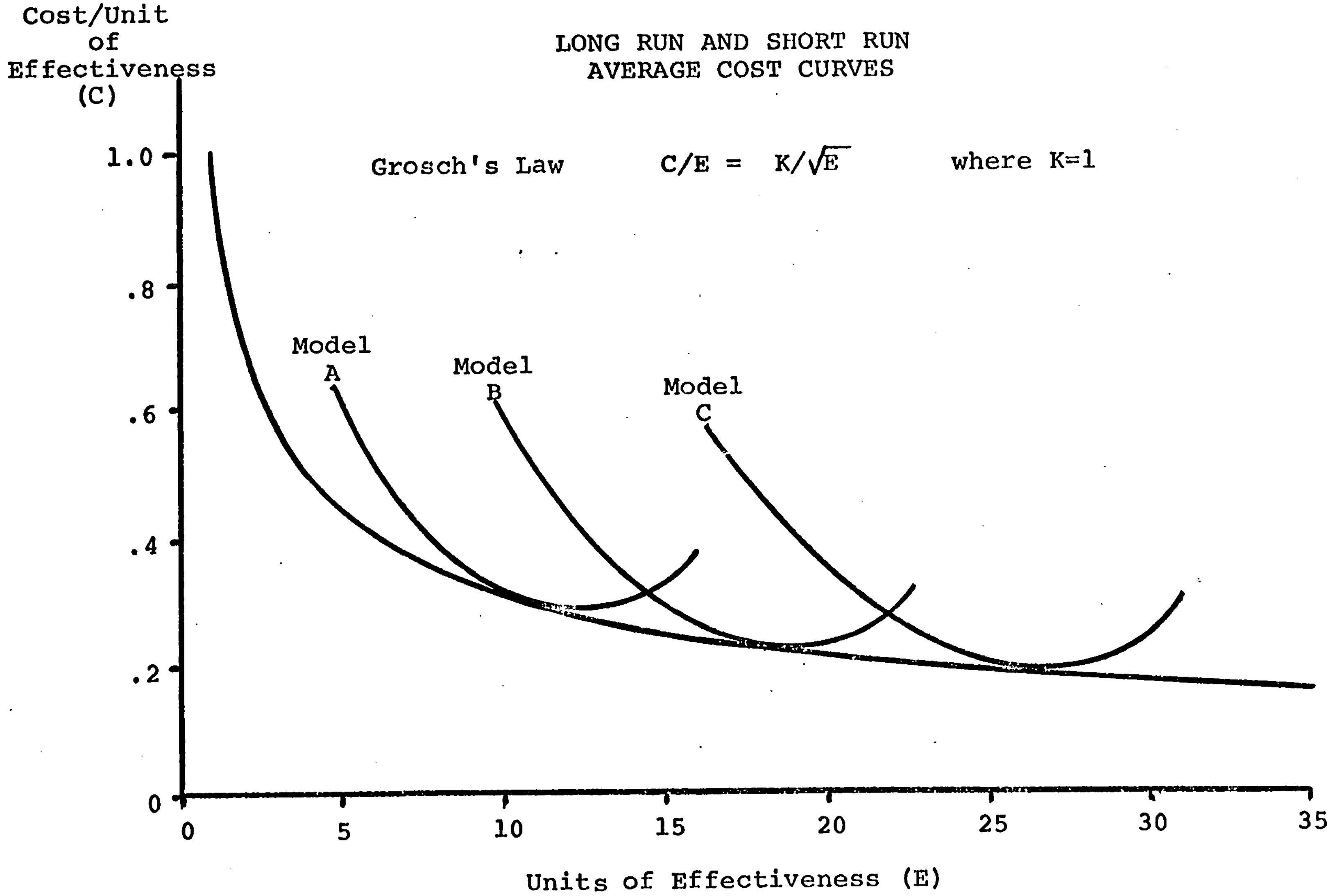
1. Whether a machine is leased or purchased. Leasing agreements often have a fixed minimum charge for so many hours of operation per month. Beyond that number of hours, there is a straight hourly charge. A purchased machine would bear the total monthly depreciation charge regardless of usage.
2. The degree to which many support costs are kept variable. For example, an in-house programming staff may be maintained, or programming services may be purchased as needed from service companies.

However, the general pattern of high fixed costs in operating a computer should produce a fairly long downward slope. Figure 1-1 illustrates these general relationships.

What is the implication of this for computer time-sharing? Suppose that 4 users each buy a $100,000 machine; their total computing power is four times that of any individual. If on the other hand, they pool their funds, buy a $400,000 machine, and work out some equitable arrangement to share it, their total computing power is now <u>sixteen</u> times that of any individual on the first plan. Each of

FIGURE 1-1

LONG RUN AND SHORT RUN
AVERAGE COST CURVES



them has at his disposal four times his previous computing power; against this he must balance any additional costs caused by the sharing arrangement (e.g., communications costs, accounting costs, etc.).

We can now see how time-sharing is able to exist in the marketplace. There are numerous reasons why time-sharing is attractive; however, if cost and effectiveness were linearly related, computer time-sharing could not be marketed at prices remotely near the prices of its current substitutes. The time-sharing vendor simply fills the "grouping" function in the example given above. As long as sales volume is kept high and the overhead and communications costs remain less than the savings on computer power, the vendor should be in fine shape.

The information contained in these last three sections should provide the necessary background for the reader who has not had experience with computer time-sharing. We will now define the objectives and methodology for this study.

1.5 The Objectives of the Study

There are two main objectives in conducting this study: the first is the development of a methodology for studying prices in the commercial computer time-sharing market, and the second is the application of this methodology to the current market. This is an empirical study, aimed at determining the extent to which non-competitive pricing can exist in a rapidly growing industry with a complex product. Because of the scarcity (and non-comparability) of any past data, this is a static analysis only; however, using the methodology developed in this study, additional comparable data points can be established at successive points in the life cycle of this young industry. Then, the development of various economic characteristics can be closely observed over time. For this reason, the development of the methodology is just as significant as the results of its first application which are given in this study.

This study is potentially beneficial (in the pragmatic sense) to two main groups: one is made up of time-sharing sellers and buyers, and the other is comprised of those who might have to oversee the industry (e.g., in the areas of antitrust or even industry regulation). This is in addition to the help it will provide to academicians who subsequently extend the analysis.

1.6 The General Methodology of the Study

The general structure of this study is fairly typical of many economic studies. As usual, a model is constructed, a hypothesis is formulated based upon the model, and the hypothesis is then empirically tested. In this study, the industry is classified in line with the basic economic models of industrial organization. Then, a relationship of industry prices is hypothesized, and this hypothesis is empirically tested.

An analysis of prices is especially difficult in this industry because there is no predictable relationship between the nominal and the effective prices. This study provides a standardized measure of effective prices by defining a "typical" quantity or batch of the product and then empirically determining the effective price that a seller charges for this unit of product. These effective prices from seller to seller are then statistically analyzed.

We will review here the general presentation of the study. Chapter II is a review of the relevant economic literature, especially the literature pertaining to oligopolistic markets. Chapter III examines the economic characteristics of the time-sharing market and determines the market model (from Chapter II) to which the actual market most closely conforms. Based upon this choice of models, the hypothesized price relationship among sellers is established. Chapter IV presents a detailed analysis of the nominal pricing structure which prevails in the time-sharing industry. Chapter V then gives the methodology developed to empirically measure the effective prices in this market. Chapter VI details the statistical analyses applied and their results. Chapter VII then presents the economic conclusions derived from these statistical results.

1.7 Summary

In this chapter, we have concentrated on two broad areas. First, we have surveyed some of the basics of computer time-sharing in order to establish a common minimum level of background information. Secondly, we have outlined the objectives and general methodology of the study. We will now proceed with the body of the study.

CHAPTER II

A REVIEW OF THE LITERATURE

2.1 Introduction

This study relies most heavily upon the concepts and procedures developed by industry researchers such as Burns, Bain, Weiss, and Caves.[1] The study is quite close philosophically to the work of Bain as indicated by the following passage in which Bain states that his goal is, "to test the theory by seeing whether its hypotheses or predictions can be confirmed or disconfirmed with factual data. Theoretical predictions are viewed only as hypotheses subject to critical testing, as is appropriate in a scientific endeavor."[2] As stated in Section 1.6, the general methodology for the study will be to: (1) examine the general economic characteristics of the industry; (2) establish a hypothesized relationship of industry prices based upon these observed characteristics; (3) empirically test our hypothesis; and (4) interpret our findings.

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[1]See, for example, Arthur Robert Burns, The Decline of Competition (New York: McGraw-Hill, 1936); Joe S. Bain, Industrial Organization (2nd ed.; New York: Wiley, 1968); Leonard W. Weiss, Case Studies in American Industry (New York: Wiley, 1967); and Richard Caves, American Industry: Structure, Conduct, Performance (2nd ed.; Englewood Cliffs, New Jersey: Prentice-Hall, 1967).

[2]Bain, Industrial Organization, p. viii.

Therefore, in this chapter, we must determine what industry or market characteristics are theoretically relevant to the construction of our price hypothesis.

As we will later see, the computer time-sharing industry is obviously a differentiated oligopoly. The problem is determining how differentiated and how oligopolistic it is. Our primary need is information which will help us to hypothesize about the relative prices in the industry rather than the absolute level. Therefore, we must determine what evidence we need to construct a hypothesis concerning price competition.

First, we will examine the concept of a market and the theory of oligopolistic competition as developed by Fellner. Then, we will again examine oligopoly theory, but from an institutional viewpoint, to obtain a set of working definitions which have been refined for use in empirical studies.

2.2 Markets and Market Structures

To the price theorist, the concept of a market is cleanly defined. Assuming that quantitative limits have been set, an industry (i.e., the sellers in a market) may be defined based upon the cross elasticities of demand for the commodities involved. "A commodity group with high cross elasticities within the group but with low cross elasticities with respect to other commodities is often said to constitute an industry."[1] The difficulty of obtaining all the necessary cross elasticities--let alone defining limits for them--makes the implementation of this approach rather impractical for the empirical researcher.

In industry studies, the definition of the market is often one of the more difficult steps. Weiss defines a market as, "all the buyers and sellers, potential or actual, of a particular good or service who deal with one another or could easily deal with one another."[2] The two essential concepts contained here are the close-substitutability of producer or seller outputs and the ability of buyers and sellers to consummate transactions; if products were not close enough substitutes or if buyers and sellers were geographically separated, we would have more than one market.[3] Weiss points out that any boundries selected in industry studies are necessarily arbitrary,[4] and Caves makes the additional point that any market definition is often not equally satisfactory with respect to both the buyer and seller sides of the market.[5]

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[1]Richard H. Leftwich, The Price System and Resource Allocation (3rd ed.; New York: Holt, Rinehart and Winston), p. 43.

[2]Weiss, American Industry, p. 2.

[3]See, also, Bain, Industrial Organization, pp. 6-7.

[4]Weiss, American Industry, p. 3.

[5]Caves, American Industry, p. 7.

Once a market has been defined as to the identities of its component elements (i.e., the buyers, sellers, and product), the next step is examining the market structure. According to Bain,

> Market structure refers to the organizational characteristics of a market, and for practical purposes to those characteristics which determine the relations (a) of sellers in the market to each other, (b) of buyers in the market to each other, (c) of the sellers to the buyers, and (d) of sellers established in the market to potential new firms which might enter it. In other words, market structure for practical purposes means those characteristics of the organization of a market that seem to exercise a strategic influence on the nature of competition and pricing within the market.
>
> The most salient aspects or dimensions of market structure are:
>
> (a) The degree of seller concentration--described by the number and the size distribution of sellers in the market.
> (b) The degree of buyer concentration--defined in parallel fashion.
> (c) The degree of product differentiation as among the outputs of the various sellers in the market--that is, the extent to which their outputs (though similar) are viewed as nonidentical by buyers.
> (d) The condition of entry to the market--referring to the relative ease or difficulty with which new sellers may enter the market, as determined generally by the advantages which established sellers have over potential entrants.[1]

The term "market structure" is not universally defined in the above manner. Some authors use it in a broader sense to describe the usual theoretical market models (e.g., monopoly, oligopoly, etc.).[2] While structural

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[1]Bain, Industrial Organization, p. 7.

[2]For example, see Paul A. Samuelson, Economics (8th

considerations as outlined above do have significant influence upon the conduct of those in the market, they are not necessarily determinants of unique conduct.[1] Bain accounts for this by providing another important concept, market conduct, which " . . . refers to the patterns of behavior that enterprises follow in adapting or adjusting to the markets in which they sell (or buy)."[2]

We should note here that the terms "market structure" and "market conduct" have a very different time connotation implicitly attached to each of them. Structure is determined at a point in time,[3] while conduct is observed over a period of time. Since this study is a static analysis, we will have to rely heavily upon the structural factors to help us in hypothesizing about the conduct factors.

2.3 The Nature of Oligopoly

In the preceding section, we gave Bain's list of the important elements of market structure. They are

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ed.; New York: McGraw-Hill, 1970), p. 460.

[1]For example, in a differentiated oligopoly situation, we could have pricing by collusion, price leadership, or several other patterns.

[2]Bain, Industrial Organization, p. 9.

[3]Note that this is not saying that market structures are volatile; in fact, market structure tends to be rather stable, especially in older industries. See Caves, American Industry, pp. 32-33.

important because "they seem to exercise a strategic influence on the nature of competition and pricing within the market."[1] Oligopoly theory gives us some insight into this importance. Therefore, we will now examine the theory of oligopolistic competition as developed by Fellner.

According to Fellner, oligopoly is competition among a "few" sellers where this fewness, in turn, means that each seller is relatively important in the market.[2] Therefore, all oligopolists must be constantly aware of their mutual interdependence and must weigh their actions in terms of the reactions of their competitors who will surely notice (and possibly respond to) any actions. This mutual interdependence can make the demand and supply functions and the equilibrium situation indeterminate in oligopoly markets.

> The markets on which it is possible to define neither demand functions nor supply functions for the individual firms are those on which specific sellers or buyers sell or buy considerable fractions of the total market volume, so that the other sellers and buyers are affected materially by what single firms are doing. . . .
>
> The oligopolist, instead of "setting up" a supply function, attempts to select a definite price to be charged and a definite quantity to be sold, which, in combination with one another, are optimal from his point of view. But the quantity he is capable of

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[1]Bain, Industrial Organization, p. 6.

[2]William Fellner, Competition Among the Few (New York: Knopf, 1949), p. 17.

> selling at any given price depends on the prices charged by his competitors, which, in turn, are appreciably affected by what price he sets. Consequently, not only does the oligopolist fail to set up a supply function, but also it is impossible to define for him a demand function from information pertaining to buyers' preferences alone. . . . Determinate equilibrium, in the usual sense, does not exist if, for the individual firm it is impossible to define either a demand function or a supply function (or both). Yet, oligopolists . . . usually have cost functions and utility functions (or their equivalents), and these set limits to what is acceptable to them, that is, to what from their point of view is preferable to going out of business. The problem of these limits can be approached with the tools of traditional value theory. These limits always are in the nature of (long-run) zero-profit points. They exclude the possibility of outcomes by which any one party suffers a loss (negative profit) in relation to the zero line determined by not concluding the deal in question.[1]

Note that cost and utility functions do set limits upon any equilibrium which may be achieved. The problem is to determine what forces lead to equilibrium within these limits.

Fellner argues that equilibrium within the limits is determined by a process quite similar to what we commonly think of as bargaining. Usually bargaining involves direct or explicit negotiation during which each party attempts to determine the most favorable terms which it can extract from the other participant. Fellner feels there is no meaningful difference between explicit and implicit bargaining in which the participants attempt to estimate (without direct negotiation) what the responses of the other party will be to various actions. That is, implicit bargaining

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[1]*Ibid.*, pp. 10-12.

occurs when:

> . . . each party tries to find out from the responses of the other parties what the ultimate consequences of its own patterns of behavior are; and each party tries to discover which of the alternative patterns of behavior results in mutual reactions that are in the nature of a tacit agreement (or convention), and are more favorable from his point of view than any other tacit agreement acceptable to the others. Such processes may be termed implicit bargaining or quasi-bargaining and the resulting state of affairs may be termed quasi-agreement. The difference between "true" agreement and quasi-agreement is that the former requires direct contact while the latter does not.[1]

As soon as bargaining occurs, the question of bargaining power arises. Fellner gives four factors as the determinants of bargaining power:

1. "Long-run consequences of violating accepted value judgements (that is, of faring too well),"
2. "The immediate political consequences of a stalemate in the relations between the parties concerned,"
3. "The ability of the parties to take and to inflict losses during stalemates,"
   and
4. "Toughness in the sense of unwillingness to yield in a range in which the other party is expected to yield if one fails to do so."[2]

Fellner also states that, in societies with fairly stable

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[1] Ibid., pp. 15-16.

[2] Ibid., pp. 24-28.

standards, accurate information about the above four factors and the zero profit levels is sufficiently known so that bargains (or quasi-bargains) are reached rather efficiently.[1]

Having presented the foregoing, Fellner states the main "suggestion" of his analysis:

> In markets of the oligopolistic kind . . . , there is a tendency toward the maximization of joint profits of the group and toward division of these profits in accordance with [the four factors of bargaining power.][2]

In other words, since the oligopolists will share the industry profit "pie," they all have an interest in sharing the largest possible pie.

> Given the quasi-agreement--or a true agreement--with respect to the division of gains, it becomes possible to describe individual market functions (demand and supply functions) for the participating firms. But these are not derived from utility functions and technological functions alone. They are derived from these functions plus the interaction of [the four factors of bargaining power.][3]

Obviously these demand and supply functions would be dependent on factors which might be rather difficult to quantify (i.e., the relative strengths of bargaining power).

> Quasi-agreements established to regulate the relations between rivals must be changed, sometimes because of far-reaching changes in relative strength, sometimes because changes in the market functions make it necessary to test the views of rivals by tentative price-setting. Consequently, the periods of established quasi-agreement will be interrupted by

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[1] *Ibid.*, p. 32. [2] *Ibid.*, p. 33.

[3] *Ibid.*, pp. 33-34.

> intervals of strength-testing (aggressive) competition; and also by periods of tentative price-setting aimed at testing the market views of rivals. Effective price-leadership may eliminate interruptions of the second kind.
>
> Furthermore, even under quasi-agreements, the tendency toward joint-profit maximization is in many cases counteracted by the circumstance that the requirements set by [the four factors of bargaining power] are better satisfied at price levels and levels of output other than those maximizing joint profits. In other words, it may in many cases be impossible to accomplish an acceptable distribution of the joint profit at the values of the relevant variables which would maximize the joint profit, while some pattern of distribution relating to a smaller joint profit may prove acceptable to all parties concerned.[1]

True joint profit maximization is probably rarely attained in practice. It is not possible to accomplish without pooling all resources and earnings.[2]

> [However,] placing all relevant variables under joint control would imply a commitment with respect to the distribution of the joint gain such as would not be acceptable *a priori* to all parties. This is especially true of those variables that require skill and ingenuity in handling (such as those directly connected with advertising, product variation, technological change, and so forth).[3]

Complete pooling of all resources can place some firms at extreme disadvantage if the pooling agreement is terminated. Also, of course, many societies have institutional barriers to such extremes of co-operation by supposedly competing firms.

Fellner's approach of quasi-bargaining and quasi-

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[1] *Ibid.*, p. 34.

[2] *Ibid.*, p. 35.

[3] *Ibid.*, p. 35.

agreements modifies considerably the traditional reaction-function approach to oligopoly theory. The meaning of reaction functions can be most easily illustrated in terms of the price leadership case. The "follower" firm forms its reaction function by choosing its optimal set of market variable values for each set of market variable values which the "leader" firm might choose.

> The transition from the "reaction-function" approach . . . to the more realistic problem of quasi-agreements may therefore be said to consist of two steps. In the first place, starting from the leadership model, we should include in our arsenal of tools reaction functions other than those defined to express followers' individual profit maximization for given values of leaders' variables. Followers' reaction functions are significant only if the leader is willing to make a selection along them. Any meaningful solution assumes that the behavior pattern of the rivals is mutually acceptable. For the same reason, the second step of the transition consists of allowing for the possibility that the reaction function of the follower may be limited to one range of potential values of the leaders' variables, to that range within which the outcome is more favorable for the follower than in other ranges. The limiting case is that of a reaction point, rather than function, and in this case the transition from the original leadership model to quasi-agreement is so complete that all features of the leadership model have disappeared. For, in this case, the "leadership" has become mutual, and mutual leadership is indistinguishable from mutual followership.[1]

Using his quasi-agreement approach, Fellner develops a theory of qualified or limited joint profit maximization since complete joint profit maximization will so rarely occur in practice. The following list gives the reasons why

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[1] Ibid., pp. 126-27.

qualified maximization will occur.

a. Unwillingness to pool resources and their earnings and to agree on inter-firm compensations, in order to maximize profits in the presence of non-horizontal cost curves (pages 191-7ff.)
b. Same unwillingness in the presence of differences between cost curves of the various firms (pages 191-7ff.).
c. Same unwillingness in the presence of product differentiation, implying unwillingness to pool brands, or in the presence of spatial differentiation, implying unwillingness to reallocate output between points with different locations, pages 191-7ff.).
d. Incompleteness of co-ordination concerning future changes in advertising, product quality, and technological methods (pages 183-91ff.).
e. Safety-margin considerations (pages 146-57ff.).
f. Long-run considerations which relate to the maximization of the present value of the enterprise, in a particular sense, more than to "profits" technically (pages 158-68ff.).
g. The adoption of cutthroat policies or the desire to avoid them (pages 177-83ff.).
h. The existence of controlling groups among owners (pages 169-74ff.).[1]

Fellner goes on tc state that qualifications (e), (f), and (h) apply to monopoly as well as to oligopoly. They reflect the fact that the normal construction of the marginal functions necessary for mechanistic profit maximization decisions does not contain all variables "pertaining to the pursuit of economic advantage."[2]

The other qualifications, since they apply to oligopoly situations only, are more relevant to us. The impact of qualification (a) is shown in Figure 2.1 in terms of two

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[1]*Ibid.*, p. 198-99.

[2]*Ibid.*, p. 199.

FIGURE 2.1[1]

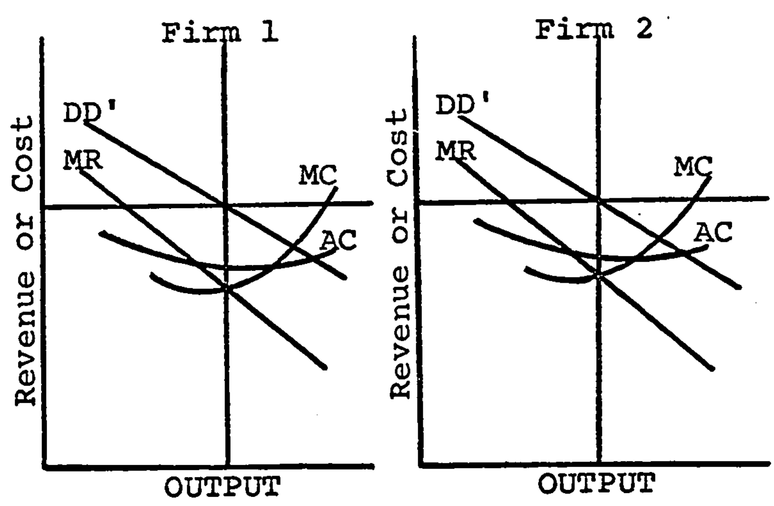
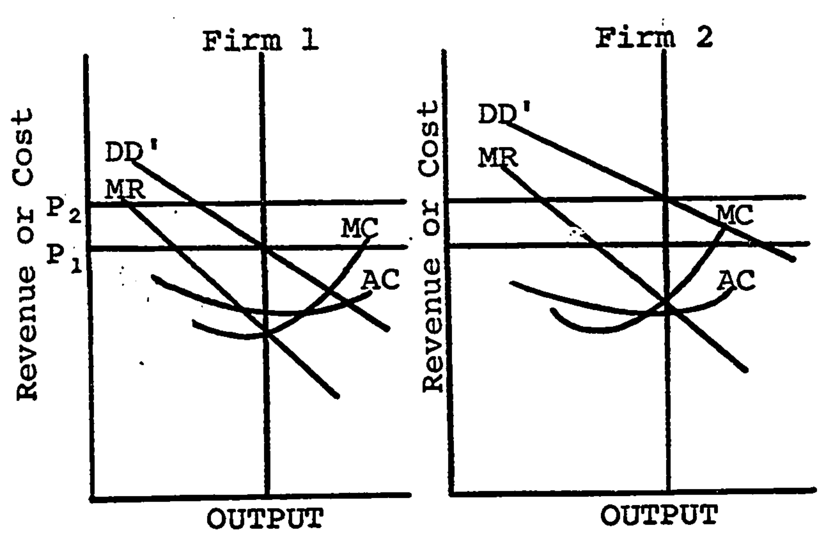


FIGURE 2.2[2]



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[1] *Ibid.*, p. 201.

[2] *Ibid.*, p. 206.

firms. Rather than the two firms having a single cost curve for any level of output (as in the case of complete pooling), there is an individual cost curve for each firm. Figure 2.1 assumes identical but sloping cost curves and definite and equal market shares. Under these assumptions, the equilibrium point is determinate since both sellers would desire the same market price. If one firm were more powerful than the other, that firm's DD' curve would have to be shifted to the right. Now, each firm desires a different price and the market price is theoretically indeterminate within the range of their desired prices. According to Fellner, the actual determination occurs either by the quasi-bargaining process or by price leadership (the pattern of which was probably determined initially by quasi-bargaining). This situation is shown in Figure 2.2 where P_1 and P_2 represent the prices desired by firms 1 and 2 respectively. The DD' curves in Figure 2.2 also assume equal market shares regardless of the market price (i.e., both DD' curves have the same price-elasticity).[1]

It is possible for the two firms to agree (or quasi-agree) upon iso-profit share rather than iso-market share demand curves. However, Fellner dismisses the iso-profit approach as being unrealistic in practical terms. The iso-profit approach means that firms must disregard market

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[1]*Ibid.*, pp. 201-06

shares. Therefore, those firms whose market shares drop to satisfy the iso-profit agreement leave themselves wide open to substantial harm if the quasi-agreement is ever abandoned.[1]

The effect of adding qualification (b) is to shift the cost curves of one of the firms upward or downward, still leaving limits such as those shown in Figure 2.2. A possibly significant impact, however, is that limits are placed upon the way in which the firms may share profits since qualification (a) allows no inter-firm compensations.[2]

The addition of qualification (c) has significantly more complex effects. "The reason is that in differentiated oligopoly the buyer has a say in the matter of market shares and in that of profit shares at equal prices."[3] Product differentiation destroys the consumer indifference which is present when we have homogeneous products and equal prices.

In Figure 2.3 (which has cost differences incorporated also), the DD' functions must be interpreted in a different manner than in the undifferentiated case.

> For oligopoly with product differentiation it is possible to assume that (1) the functions, throughout their course, imply such price ratios between the products in conjunction with such advertising expenditures by the rivals and with such product qualities as will establish iso-market-share DD' curves; (2) the functions, throughout their course, imply such price ratios

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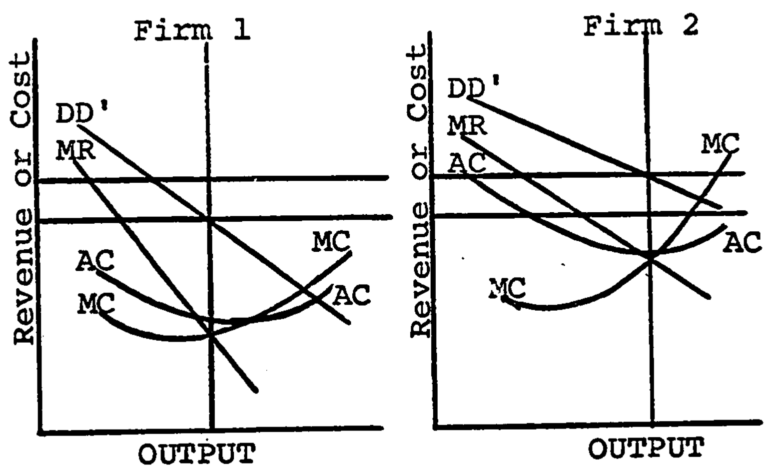
[1] *Ibid.*, p. 217. [2] *Ibid.*, pp. 209-10.

[3] *Ibid.*, p. 213.

between the products in conjunction with such advertising expenditures by the rivals and with such product qualities as will establish iso-profit-share DD' curves in accordance with relative strength; (3) the functions, throughout their course, imply a constant (absolute or proportionate) price ratio between the products and given advertising and product quality. In all three cases the unwillingness to pool--in the sense of qualification (c)--expresses itself in the fact that brands exist if a producer finds it profitable to produce them, given the brands of other producers, and not only if the sum of the profits is increased by the separation of markets which the existence of the brands makes possible.[1]

FIGURE 2.3[2]



Again, Fellner feels that the iso-profit approach (i.e., the second interpretation) is unrealistic for the same reasons given earlier. He goes on to state that:

It should be emphasized that the sharing agreement

[1] Ibid., p. 213. [2] Ibid., p. 212.

> (market- or profit-sharing) need not relate directly to sharing in any of the three cases just distinguished, that is, in any of the cases under product differentiation. A quasi-agreement relating to price ratios, advertising policy, and product variation policy *is* implicitly a sharing agreement.[1]

There are certainly more elements of conflict to be resolved by quasi-agreement when product differentiation is involved. Instead of simply agreeing to a point along the iso-market share curves, agreements must also be reached as to price differences, advertising, and quality.

If we incorporate qualification (d), we see that market shares can change substantially over time. This situation is typified by "non-price competition." A quasi-agreement exists which regulates price differentials *only*. The effect of such non-price competition can vary according to its intensity and the quality elasticity of the product. In describing non-price competition, Fellner says:

> Conditions of this sort can exist only if there is serious disagreement concerning the evaluation of relative skills. Disagreement is likely to arise because we are dealing here with persons who have chosen a profession primarily suitable for those who believe that they can outdo their rivals. Businessmen do not probably believe that they are more skillful than their rivals in cutting the price; hence price warfare is not a normal feature of oligopolistic markets, except when it is necessary to test relative standing power. But businessmen *are* apt to believe that they are more skillful than their rivals in such things as technological and organizational improvement, product variation, and advertising; hence they may never end testing their skills in these respects. Yet non-price competition on the assumption that the rival is completely uninfluenced

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[1] *Ibid.*, p. 213.

> or impotent is an extreme case. Limited non-price competition which restricts itself to certain methods of improvement and of new advertising is more likely, especially if it is felt that these methods are tied more closely to certain "skills."[1]

The last qualification which is unique to oligopoly is (g). According to Fellner, "The desire to avoid cut-throat competition tends to make for downward price rigidity and for upward price flexibility."[2] Since Fellner's diagrams illustrate quasi-agreement, they are not capable of showing cutthroat competition.

Analysis of oligopolistic industries can be further complicated by the presence of non-oligopolists in the market. Several possible relationships may exist between the oligopolists and any atomistic competitors which may be present. For example, the dominant firms might force the terms of their quasi-agreement upon the atomistic firms. However, this enforcement might be difficult; the dominant firms may not be able to exert pressure selectively upon particular atomistic competitors.

Fellner suggests that it is more likely that the large firms will possess only partial oligopoly power. In partial oligopoly, the large firms act as oligopolists while taking for granted competitive behavior on the part of their atomistic competitors. If either entry restrictions, cost

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[1]*Ibid.*, p. 221.

[2]*Ibid.*, p. 227.

disadvantages for new entrants, or product differentiation curtail the entrance of additional atomistic competitors or place them at a significant cost disadvantage, then we might expect partial oligopoly. The small firms are not able to exert sufficient competitive pressure to drive prices down to the zero profit level for all atomistic competitors, let alone for all firms. Therefore, the large firms may find it more profitable to allow these atomistic competitors to continue to exist as opposed to lowering prices enough to drive them out of the industry. As noted above, however, this analysis depends upon certain assumptions which limit the amount of competitive pressure that atomistic competitors can exert.[1]

This summary of Fellner's analysis of oligopolistic competition gives us a theoretical framework which we can employ to examine and describe the process of pricing in an oligopolistic market. In our analysis of the time-sharing industry, we are going to have to rely heavily upon institutional details for many clues as to the degree of competition which exists in the industry. Therefore, a review of the institutional aspects of oligopoly will provide us with a set of concepts and definitions which have been refined through successive applications in studies of other industries. As we examine this institutional material, we

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[1] *Ibid.*, pp. 136-41.

will tie it as closely as possible to our work with Fellner's model.

2.4 Seller and Buyer Concentration

As defined earlier, seller and buyer concentration refer to the number and size distribution of sellers and buyers in the market. If viewed from either side, we may vary from one participant (monopoly or monopsony) to innumerable participants (atomistic competition). When a few firms in an industry share a high percentage of the sales, we speak of the industry as being "concentrated." According to Adelman, "there is no logical connection between concentration and any behavior pattern in any instance. But, as a general statistical matter, the greater the concentration the lower the odds in favor of competitive behavior."[1] In terms of the Fellner model, the degree of seller concentration is the determinant, of course, of the degree of mutual interdependence among the sellers.

Buyer and seller concentration are, of course, very similar concepts; however, buyer concentration seems a much less important problem, both from the standpoint of its actually arising and from the attention it has received in the literature. Although we will see in Chapter III that

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[1]M. A. Adelman, "Industrial Concentration" (a statement to the Subcommittee on Antitrust and Monopoly, U. S. Senate Judicial Committee, September 10, 1964), p. 7.

various buyer characteristics are important in our analysis, the problem of buyer concentration (i.e., monopsony or oligopsony) does not arise. Therefore, we will ignore it also here in Chapter II.

An obvious, practical problem in empirical studies is the determination of the exact point at which the mutual interdependence of sellers becomes perceptible. Everyone will agree that a situation in which three sellers have a reasonably equal share of the market is clearly an oligopoly market of some type. Similarly, if 500 sellers had equal shares of their market, everyone would agree that was some variety of atomistic competition. However, there is no accepted dividing line between these two cases.

Recognizing the difficulty of applying any precise quantitative standards to the problem of classifying industries according to theoretical market types, Bain has instead approached the problem by attempting to state some useful general relationships.

> First, other things being equal, oligopolistic interdependence becomes stronger as seller concentration becomes higher, or weaker as seller concentration is less. Second, therefore, the higher the degree of seller concentration within oligopoly, the greater is the probability of the adoption of joint monopoly price and output policies by rival sellers.[1]

This is essentially a restatement of Fellner's main theme. Bain goes on to concisely describe the conflicting desires

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[1]Bain, Industrial Organization, p. 117.

facing every oligopolist.

> In sum, there will be in the usual oligopoly a counterpull between joint profit-maximizing and independent profit-maximizing motives. . . . in this connection, two specific aspects of the degree and pattern of seller concentration should influence the comparative importance of independent and joint action within oligopolies:
>
> 1. The degree of seller concentration, as reflected in the number of sellers and the proportions of the whole market supplied by individual sellers.[1]
> 2. The presence or absence of a competitive fringe of small sellers in an oligopolistic industry, and its quantitative importance if it is present.[2]

As we will later see, the second point may be especially relevant to this study. Even though these fringe sellers may be seemingly unimportant in terms of market share, they may exert pressure upon the dominant firms in the industry. For example, unless the dominant firms enjoy a considerable cost advantage, they may be motivated to price lower (than they otherwise might) in order to prevent gradually increasing market penetration by the smaller firms. On the other hand, the smaller firms might find it advantageous to remain under the pricing "umbrella" of the larger firms, not attempting to penetrate to the point where the larger firms would respond to the threat. This is similar to Fellner's analysis of partial oligopoly cases.

Caves makes the interesting point that, "concen-

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[1] *Ibid.*, p. 120.

[2] *Ibid.*, p. 122.

tration by and large rests on and reflects other elements of market structure."[1] In other words, concentration is not a phenomenon which has merely randomly arisen in certain industries and not arisen in others. In virtually all cases, the level of concentration is quite predictable in terms of the product differentiation and entry barriers which are present in a particular industrial situation.

2.5 Product Differentiation

According to Chamberlin, product differentiation may be defined as follows:

> A general class of products is differentiated if any significant basis exists for distinguishing the goods (or services) of one seller from those of another. Such a basis may be real or fancied, so long as it is of any importance whatever to buyers, and leads to a preference for one variety of the product over another. . . .
>
> Differentiation may be based upon certain characteristics of the product itself, such as exclusive patented features; trade-marks; trade names; peculiarities of the package or container, if any; or singularity in quality, design, color, or style. It may also exist with respect to the conditions surrounding its sale.[2]

While Chamberlin's definition is equally valid for both the consumers' goods and producers' goods markets, significant product differentiation is much more prevalent in consumer

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[1]Caves, American Industry, pp. 17-18.

[2]E. H. Chamberlin, The Theory of Monopolistic Competition (Cambridge: Harvard University Press, 1933), p. 56.

markets. Since this study involves a producers' good, we must look fairly carefully at the various sources of product differentiation so that we may later examine each for relevancy to our problem.

According to Bain, the major sources of product differentiation are: (1) differences in quality or design among products; (2) the inability or unwillingness of buyers to evaluate the important characteristics of the goods they are buying; (3) the persuasion of buyers by advertising and other sales promotion activities; (4) the attachment of an image of status or prestige to particular goods; and (5) differences in seller location.[1] According to Caves, product differentiation is especially important in situations such as the following:

> Where the product fills no simple technical function, but rather can satisfy many different sorts of personal needs or uses, psychic or physical, consumers probably will have different preferences among brands. Many drug and cosmetic articles would fall into this class. Where consumers typically lack the skill to evaluate different brands, they may form their preferences on the basis of superficial appearance, advertising claims, and the like. This case includes many ordinary consumers' durable goods. A household purchases them relatively seldom, and thus can form no judgement of the exact merits of different brands from its own experience. The result, again, is likely to satisfy all the conditions necessary for product differentiation. Finally, some goods and services are innately complicated, and thus can satisfy or disappoint consumers' expectations in many different ways. Consumers will value each brand's pluses and minuses differently, and thus come up with diverse preference

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[1]Bain, Industrial Organization, pp. 226-27.

patterns.[1]

On the other hand, producers' goods are bought to be used as inputs to the production process; the criteria for utility evaluation should be relatively free of such things as complex psychic motivations (e.g., status or fulfilling childhood desires). Also, talent should be available to perform analyses upon prospective purchases to determine the best buy on a cost-benefit basis. Caves gives the following example drawn from the producers' goods market:

> Even where physical differences exist, no economic differentiation may arise if the buyers can make an exact appraisal of the differences and if every buyer makes the same appraisal. For instance, coal of different grades and from different regions may vary in energy content and in the quantities and types of impurities contained in it. Major users of coal, however, such as giant electrical utilities, can readily measure these differences. They may decide that type A coal has exactly 1.3 times the energy value to them per ton as type B coal. And then they will be willing to pay more for type A than type B. But this is not the same as product differentiation. The utilities will pay more for type A, but only 1.3 times the price per ton of B, and not a penny more. If the price relation between A and B should vary from this standard differential, all utilities would switch from one to the other. The market result is the same as if the product were undifferentiated.[2]

The above example is extremely relevant to this study. As we will see in the following chapters, this study makes exactly the same type of analysis relative to time-sharing

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[1]Caves, American Industry, p. 21.

[2]Ibid., p. 21.

and attempts to allow for physical differences in the product, just as described above.

The concept of product differentiation as stated, for instance, by Chamberlin is quite acceptable until one attempts to apply it in practice. It is true that, "After all, no real economic difference exists between varying the traits or qualities of the product and varying the advertising which promotes it. Either tactic simply adjusts the blandishments with which the seller tries to woo the consumer."[1] However, when we are attempting to hypothesize about price equality, we cannot help but wish that we had a conceptual definition of meaningful vs. nonmeaningful differences in product. Such differences could only have meaning though in the producers' goods market where we can apply normative standards to the buyers' motivations and selection criteria. To put this another way, any imperfections in the substitutability of products in the producers' goods market should be explainable by differences in the productivity of the products.

What is the effect of product differentiation upon the price relationships in an industry? According to Brennan:

> . . . differentiation permits oligopolistic firms in the same industry to charge different prices. For if advertising, product design, and other selling activ-

[1]*Ibid.*, p. 21.

> ities can persuade consumers that a certain brand has advantages over others, the firm selling that brand will be able to command a higher price. Slight price differences can exist even under informal collusion; the price leader sets his price and others set theirs very close to it. The result is that there will be not one equilibrium market price but an equilibrium cluster of prices. Then changes in market demand, in factor prices or technology that affect all firms in the industry, will cause the entire cluster to rise or fall. There remains, however, the tendency toward price rigidity.[1]

Although Brennan's conclusions about differing prices are given in terms of a differentiated oligopoly, they would also apply to monopolistic competition.[2] Bain adds a second possibility. In a differentiated market, there may arise two distinct groups of sellers: one group which enjoys a higher position in almost everyone's preference patterns and a second group which is therefore forced to compete by selling at lower prices.[3] The concept of one or more clusters of prices is perfectly logical in terms of Fellner's analysis. A cluster of prices would be a natural offshoot of a quasi-bargaining approach among a group of firms which have roughly similar approaches to the

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[1]Michael J. Brennan, Theory of Economic Statics (Englewood Cliffs, New Jersey: Prentice-Hall, 1965), pp. 257-58.

[2]Brennan has a rather low opinion (shared by this author) of the practical usefulness of the concept of monopolistic competition. See, ibid., pp. 266-67. For an interesting retrospective view of monopolistic competition, see George J. Stigler, The Organization of Industry (Homewood, Illinois: Irwin, 1968), pp. 309-21.

[3]Bain, Industrial Organization, pp. 230-31.

market (e.g., fairly similar degrees of product differentiation). Multiple clusters could similarly occur when there are two or more distinct groups of firms having different approaches to the market. The quasi-agreements in the latter case would occur both within and between clusters.

2.6 The Condition of Entry

The third major element of market structure is the condition of entry to the market. "Just as concentration reflects the number of actual market rivals of a firm, so the condition of entry tells the story of potential rivals."[1] According to Bain, who has written the classic work in this area, "The condition of entry may be evaluated by the extent to which established sellers can persistently raise their prices above a competitive level without attracting new firms to enter the industry."[2] The concept of entry contains two distinct elements. "An addition to industry capacity already in use, plus emergence of a firm

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[1]Caves, American Industry, pp. 22-23.

[2]Joes S. Bain, Barriers to New Competition: Their Character and Consequences in Manufacturing Industries (Cambridge: Harvard University Press, 1956), p. 5. Stigler bases his definition upon costs instead. "A barrier to entry may be defined as a cost of producing . . . which must be borne by a firm which seeks to enter an industry but is not borne by firms already in the industry." See, Organization of Industry, pp. 67 ff.

new to the industry, are . . . required."[1] The most significant barriers to entry are: (1) absolute cost advantages of established firms over potential entrant firms; (2) product differentiation advantages of established firms over potential entrant firms; and (3) significant economies of large-scale operation.[2]

Bain presents in tabular form an elaboration of the above three points:

> I. Typical circumstances giving rise to an absolute cost advantage to established firms.
>
> A. Control of production techniques by established firms, via either patents or secrecy. (Such control may permit exclusion of entrants from access to optimal techniques, or alternatively the levying of a discriminatory royalty charge for their use.)
>
> B. Imperfections in the markets for hired factors of production (e.g. labor, materials, etc.) which allow lower buying prices to established firms; alternatively ownership or control of strategic factor supplies (e.g. resources) by established firms, which permits either exclusion of entrants from such supplies, driving entrants to use inferior supplies, or discriminatory pricing of supplies to them.
>
> C. Significant limitations of the supplies of productive factors in specific markets or sub-markets for them, relative to the demands of an efficient entrant firm. Then an increment to entry will perceptibly increase factor prices.
>
> D. Money-market conditions imposing higher interest rates upon potential entrants than upon established firms. (These conditions are apparently more likely to be effective

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[1] Ibid., p. 5.

[2] Ibid., p. 12.

as a source of advantage to established firms as the absolute capital requirement for an efficient entrant increases.)

II. Typical circumstances giving rise to a product differentiation advantage to established firms.

A. The accumulative preference of buyers for established brand names and company reputations, either generally or except for small minorities of buyers.

B. Control of superior product designs by established firms through patents, permitting either exclusion of entrants from them or the levying of discriminatory royalty charges.

C. Ownership or contractual control by established firms of favored distributive outlets, in situations where the supply of further outlets is other than perfectly elastic.

III. Typical circumstances discouraging entry by sustaining significant economies of the large-scale firm.

A. Real economies (i.e. in terns of quantities of factors used per unit of output) of large-scale production and distribution such that an optimal firm will supply a significant share of the market.

B. Strictly pecuniary economies (i.e. monetary economies only, such as those due to the greater bargaining power of large buyers) of large-scale production, having a similar effect.

C. Real or strictly pecuniary economies of large-scale advertising or other sales promotion, having a similar effect.[1]

In reviewing the results of one of Bain's studies which found that the liquor, cigarette, automobile, quality fountain pen, and farm tractor industries had quite high barriers to entry, Caves makes the following comment:

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[1]*Ibid.*, pp. 15-16.

> But, contrary to what you might expect, scale economies explain none of the really high barriers. Here, product differentiation plays the leading role. The man with the new and better mousetrap finds not the world beating a path to his door, only the postman with enormous bills from his advertising agency.[1]

In fact, Caves has the rather strong feeling that, except in industries where the product is perfectly homogeneous, product differentiation is the major source of barriers to entry.[2] However, despite his comment above, this product differentiation could arise from some source other than advertising (e.g., product improvement through research and development).

2.7 Other Elements of Market Structure

Caves presents three additional elements of market structure; the growth rate of market demand, the price elasticity of market demand, and the ratio of fixed to variable cost in the short run. However, he does definitely assign less importance to these three elements compared to the three we have discussed.[3]

As to the effect of industry growth, Caves says the following:

> Firms in the fast-growing industry will see high profits in the offing if they increase their individual shares of the market. Even if cutting the price or

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[1] Caves, American Industry, p. 29.

[2] Ibid., p. 33.

[3] Caves, American Industry, p. 16.

> raising the quality of the product sacrifices profits this year, the returns from having a bigger share of next year's bigger market may more than compensate for this year's profit reduction. Firms in the fast-growing industry are likely to be highly competitive in their behavior.[1]

This analysis above is especially relevant to the case of the differentiated oligopoly.

Again assuming a differentiated oligopoly, the prospect for price competition is more promising in the case where there is a tendency toward elasticity for industry demand. In this case, cuts in price by individual firms will tend to increase the demand for the industry as a whole; therefore, increases in sales are not totally at the expense of other firms in the industry.[2]

A high ratio of fixed to variable costs in the short run can exert a strong pressure for price cutting in an effort to cover these fixed costs. In other words, if the variable cost of producing an item is low, prices can be reduced substantially below the long run-acceptable level and still provide some recovery of fixed cost.

2.8 Summary

Our purpose in Chapter II has been to review the economic theory and the related institutional topics which we need to establish our main hypothesis for the study.

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[1] *Ibid.*, p. 31.

[2] *Ibid.*, p. 31.

Therefore, we have concentrated on those factors which may have some value in predicting the level of price competition in an industry. Obviously, many related topics have been omitted such as the social implications of various forms of industrial organization and a review of the multitude of theoretical market models unrelated to this study. The theory contained in these latter topics does not aid us in achieving the objectives of the study.

In Chapter III, we will attempt, where possible, to examine elements of market conduct. However, market conduct is, in general, an output of this study--not an input to it. Many questions about market conduct cannot be answered by this study alone; the answers to these questions (e.g., is there collusion--either tacit or express--in the pricing policies over time?) will require a series of such studies performed over time.

This is the main body of theory contained in the study. Some additional areas of theory will be introduced as they arise so that they can be tied more closely to the related empirical observations.

CHAPTER III

THE COMPUTER TIME-SHARING INDUSTRY

3.1 Introduction

We must now examine the structure of the computer time-sharing industry in a search for clues which will allow us to hypothesize the price relationships which might exist in the light of the theory developed in Chapter II. However, before examining the elements of structure, we will look briefly at the history of the market, the pricing structure which has evolved, and the buyer motivations and alternatives. Then we examine the structure of the market in detail and end the chapter by stating the basic hypothesis of this study.

3.2 A Brief History of the Time-Sharing Market

The time-sharing market is quite young, being now only in its fifth year. Even the very concept of computer time-sharing is not much older. A British mathematician, Christopher Strachey, is generally credited with being the first to meaningfully advance the concept in a paper given in 1959 at a UNESCO International Conference on Information Processing.[1] The first brief demonstration was in November,

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[1]T. James Glauthier, "Computer Time-Sharing: Its

1961, at the M.I.T. Computation Center, and, by September of 1962, Bolt, Beranek & Newman also had a primitive system in operation.[1] In November of 1963, Project MAC[2] was established at M.I.T. with a 3 million dollar annual budget supplied by the Advanced Research Projects Agency.[3] Systems such as Project MAC--using IBM equipment--and a subsequent project at Dartmouth--using GE equipment--provided the early proving grounds for general purpose time-sharing systems.

Although there were some tentative earlier offerings, 1965 marked the real beginning of commercial computer time-sharing. Among the early firms were KEYDATA; Bolt, Beranek & Newman; IBM; and Computer Sciences Corporation. From 1965 on, growth was rapid; the concept had proven to be commercially feasible.

Origins and Development," COMPUTERS and AUTOMATION, October, 1967, p. 24.

[1]Ibid., p. 24.

[2]The acronym MAC has been stated by various sources as standing for multiple-access computer, machine-aided cognition, and man and computer--take your pick! An excellent description of Project MAC is given by R. M. Fano and F. S. Corbato in "Time-sharing on Computers," Scientific American, September, 1966, pp. 129-40.

[3]Jeremy Main, "Computer Time-Sharing--Everyman at the Console," FORTUNE, August, 1967, p. 91.

3.3 The Problem of Nominal vs. Effective Prices

The time-sharing industry has developed a price structure which makes price comparisons extremely difficult. Computer time-sharing, like numerous other services, is priced in terms of "producer effort" rather than in terms of some actual accomplishment which the buyer might be able to evaluate much more easily. For example, a physician charges $10.00 for a visit or a business consultant charges $100 per day, while a dry cleaner charges $1.50 to clean and press a suit or a barber charges $3.00 for a haircut. Computer time-sharing is priced in a manner more like the services of the physician or the consultant. The customer may know the amount of effort to be expended in his behalf while remaining very uncertain as to the benefit he will receive from this expended effort.

In the normal economic transaction (i.e., the mutually willing exchange of positive goods), each party gives a good and receives a good with his participation predicated upon an expected gain in utility. In the typical time-sharing transaction, there is no particular problem in analyzing the flow of goods from the buyer to the seller--it is money. The other flow of goods--from seller to buyer--is the interesting one in this case.

The seller of time-shared computer facilities usually prices his product in some way similar to the following:

1. A charge per hour for each hour that the buyer is connected by telephone to the system (i.e., connect charge).

2. A charge for each second that the computer's central processing unit (CPU) actually works on the buyers program (i.e., CPU charge).

3. A monthly charge for each unit of auxiliary storage which the buyer's programs or data files require (i.e., storage charge).

Some systems use a somewhat more complex pricing model, but this will suffice for the example. The important thing to realize here is that nothing is specified as to what the seller's computer system can do in a connect hour or in a CPU second. Further, there are no standards, formal or informal, by which system performance is widely measured or quoted.

This situation is further complicated because these quoted prices/"unit" vary over such a wide range.[1] Connect charges can range from nothing to $30 per hour, and CPU

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[1]The president of Remote Computing Corporation delivered a rather scathing indictment of the current level of sophistication employed in pricing by this industry. His main point is that currently most firms have such terrible data that they have no idea whether their prices reflect costs. See Joseph T. Hootman, "The Pricing Dilemma," DATAMATION, August, 1969, pp. 61-66.

charges can range from nothing to $.50 per second.[1]

The buyer obviously does not really need connect hours or CPU seconds; he needs the answers to specific problems or the output of specific programs. The raw power of the computer is strictly a means to an end for the buyer. The buyer must somehow bridge this conceptual gap between the seller's offerings and his own needs.

In this study, we will refer to these vendor price structures as the "nominal" prices. These nominal prices convey little information and serve only as the input to price analyses. We will refer to the calculated prices (i.e., the results of our price analyses) as the "effective" prices. These effective prices are our estimates of the true selling price (and buyer cost) of computer time-sharing. In Chapters IV and V, we will examine in detail the analytical and experimental procedures necessary to convert nominal prices to effective prices.

3.4 Buyers' Preferences and Alternatives

Before attempting to define the computer time-sharing market, we will examine the motivations of time-sharing buyers and the alternatives open to them.

There are two situations in which time-sharing is the relatively unique answer to a buyer's needs: first, if

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[1] "General Purpose Time-Sharing Survey," COMPUTERWORLD, March 25, 1970, pp. 56-58.

there is a need for a very short turnaround time, and, second, if there is a need for a significant level of interaction.[1] If either of these needs are present, time-sharing is the obvious answer. A large firm may then have the alternative of setting up time-sharing on its own computer; however, this alternative is not open to the firm which needs some time-sharing but not enough to install its own facilities.

The purchase of time-sharing can also be required if a time-sharing firm has in its library a truly unique program or set of programs which a particular buyer needs. In this case, the buyer's only alternative may be a massive, extremely costly programming effort.

Much of the work which is done on time-sharing could be done relatively easily using other methods. We are not saying that time-sharing is not the best way to do it--just that there is nothing making time-sharing a particularly unique method in these cases. Some of the factors which might motivate this type of buyer are:

1. a need for extra computer capacity on a temporary basis. Computers are quite "chunky" to purchase and small increases in capacity are usually inefficient to obtain.
2. a desire to avoid the managerial headaches

[1]See Section 1.2 for elaboration.

and/or start-up costs of operating an in-house computer.

3. a need for occasional usage of large computer facilities while satisfying routine needs upon a small machine.
4. a desire to control costs more closely than is currently possible for many firms since "unbundling" has taken place.
5. An unwillingness to have financial commitments in the computer area any longer than absolutely necessary.

None of these above reasons are peculiar to time-sharing only. For instance, the use of batch processing by a service bureau would meet all of them moderately well. To the extent that this is true, time-sharing must be cost-competitive with the service bureau for that particular application. If, of course, the unique talents of time-sharing are required, the two alternatives must still be cost-competitive; however, heavy economic costs are now imposed upon the service bureau because of its shortcomings in the particular application.

In general, the substitutes for commercial time-sharing are:

1. Manual or semi-manual computations. This is increasingly attractive in some applications

as electronic calculators become more sophisticated.

2. In-house processing, either batch or time-shared.

3. Service bureau processing, either batch or remote batch.

The degree of cross price elasticity with respect to each alternative can be determined only upon examination of the particular application. Little if any work has been done in this area.[1]

3.5 The Time-Sharing Market

The term computer time-sharing could have various meanings to different readers. For purposes of this study, commercial computer time-sharing firms will be defined as those which deal primarily in interactive computer power accessed through low speed remote terminals. The provision of other services (e.g., remote batch processing) does not eliminate a firm from consideration so long as its primary aim is the sale of time-sharing.

In addition, only firms capable of servicing a

[1]Surprisingly little work has even been done on the more basic question of evaluating the relative efficiencies of batch processing and time-sharing for various applications. For an exception, see: Michael M. Gold, "Time-Sharing and Batch Processing: An Experimental Comparison of Their Values in a Problem-Solving Situation," Communications of the ACM, May, 1969, pp. 249-59.

fairly broad segment of the market will be considered. There are some firms which specialize in accommodating users whose demands are very minimal; these firms must be considered as being out of the market mainstream. In reality, these firms constitute a sub-market of sorts and are segmented from the main market by both economic and technical factors. For example, they cannot accommodate many typical sized programs or perform significant computational tasks in reasonable times; however, they are extremely inexpensive for the user who runs only small, short programs.

These restrictions are not especially stringent. In fact, they would disqualify few firms which consider themselves to be in the commercial time-sharing market.

There is a real shortage of factual information about the current time-sharing market. The primary reasons are the relative newness of the industry, the lack of an active and effective trade association, and the nature of the firms. With respect to the latter, some of the firms are divisions of large companies and their performance is hidden in the maze of corporate accounting. On the other hand, the smaller firms are often closely held and, therefore, are equally hard to analyze. Despite these problems, informed observers have drawn some conclusions about the industry in general and some firms in particular.

To give some perspective to the time-sharing market, let us look at a few statistics for the computer industry in general. The total computer industry--hardware, software, and services--had sales of 10.1 billion dollars in 1969, a 17 per cent increase over 1968. The sales of major computer equipment however remained at approximately 7 billion dollars for both years; the growth actually occurred in the service area, especially independent software suppliers (66 per cent) and on-line computer services (67 per cent).[1]

A significant portion of the on-line computer sales are in the area which we have defined as computer time-sharing. Time-sharing sales have reached 150 million dollars in just six years,[2] and projections of a 2 to 5 billion dollar market by the mid-1970's are common (though perhaps optimistic). The exact number of firms sharing the current market is anyone's guess, but it is probably well in excess of 200 firms. In the March 25, 1970, issue of COMPUTERWORLD, a weekly computing industry newspaper, a list of 126 firms currently marketing time-sharing in the United States was given with no claims made that the list was anywhere near exhaustive.[3] No meaningful estimates are

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[1]"Industry Revenue Rises 17% to $10.1 Billion in 1968," COMPUTERWORLD, March 11, 1970, p. 41.

[2]Drake Lundell, "Why the Shakeout?," COMPUTERWORLD, March 18, 1970, p. 54.

[3]"General Purpose Time-Sharing Survey," pp. S6-S8.

available on the total number of buyers in the market.

3.6 Seller Concentration and Other Seller Characteristics

The president of one of the larger independent[1] time-sharing companies, Robert F. Guise, Jr., of Com-Share, Inc., gave the following estimates of approximate market share during an interview:[2]

1968

Firm	% of Market
General Electric	40%
Service Bureau Corp.[3]	17%
Next four firms	25%
Remaining fifty firms	18%

1969

Firm	% of Market
General Electric	35%
Service Bureau Corp.	20%
Next six firms	25%
Remaining seventy firms	20%

[1]Time-sharing companies are said to be independent when they are not affiliated with a computer manufacturer.

[2]Phyllis Huggins, "Guise Reviews T/S User's Buying Habits," COMPUTERWORLD, March 25, 1970, p. 49.

[3]The Service Bureau Corporation (SBC) is a wholly owned subsidiary of IBM.

These figures are not advanced here as fact; they may be the result of very informal survey techniques. Nevertheless, they give some idea as to the distribution of market shares in the industry.

A cursory examination of these market share figures would seem to indicate a quite high degree of seller concentration since the leading two firms, GE and SBC, have 55 per cent of the nationwide market. Moreover 80 per cent of all sales are accounted for by only eight firms.[1] However, there is an important factor to be considered here; these figures are for nationwide sales and do not necessarily reflect the degree of concentration existing at local levels. This market is somewhat similar to the beer market in this respect. A few firms are nationwide marketers and compete among themselves all over the country. At the same time, they have many more competitors who are only regional in geographic scope.

In the time-sharing market, the two dominant firms, GE and SBC, are nationwide competitors; however, in any given geographic region, they have other competitors who are less than national in scope. The difficulty of defining a market is increased because the national market is not divided into several precisely defined regional markets.

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[1]In the computer market, seller concentration is quite pronounced. IBM currently has approximately 70 per cent of the sales in that market.

Instead, the regional firms are based in numerous cities and extend their geographic coverage in varied directions.

While the geographical patterns involved make the precise definition of a market difficult, one industry pricing policy makes it less necessary. Although communications costs are not inconsequential, the firms in this industry price with a "freight absorption" policy. In other words, wherever a firm elects to market, it does so at the same prices that it charges in all other areas in which it markets.[1] The technical term for this is "zone pricing" with a single zone.[2] The effect upon our analysis is to ease the necessity of having to precisely define the market in geographic terms; however, the market must still be precisely defined in any other sense.

Observations of and conversations with independent sellers in this market do not indicate to the author that these sellers feel constrained by the presence of the "dominant" firms in their market. This is not true in the

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[1]Note though that a buyer who wishes to buy but who is located in an area in which the vendor does not normally market pays the communication costs to the vendor's nearest normal outlet.

[2]For a discussion of the price discrimination implicit in zone pricing, see: Burns, Decline of Competition chap. VI-VII; and Dudley F. Pegrum, Public Regulation of Business (rev.ed.; Homewood, Illinois: Irwin, 1965), chap. 9. A general conclusion of both is that this pricing scheme may be socially desirable when local markets cannot sustain efficient size plants. This is certainly the case in the beginning stages of this industry.

general computer market; there, the dominance of IBM is a fact of life and everyone acts accordingly. A reasonable statement would be that the time-sharing industry leaders are respected but not overly feared.

The firms currently in the market seem to fall into five general categories as far as their origin and orientation to the market is concerned:

1. Subsidiaries or divisions of major hardware manufacturers. The time-sharing subsidiary can serve as a "laboratory" as well as a means of starting new customers on the parent's particular brand of equipment.
2. Service bureaus which have extended their traditional services to include this additional item. Most of these firms can accommodate a broad variety of customer computer needs.
3. Firms which have a specialized software package and sell computer time-sharing as an implementation medium for this software. For example, a firm might offer a management information system or an engineering package in this manner.
4. Firms which are primarily in other areas but which have idle computer time and/or specialized software which they wish to sell.
5. The "normal" independent time-sharing firm

> established and operated with the primary aim of selling time-shared computer time. Normally such a firm has no particular "gimmick" but relies instead upon the growing market.

Certainly some firms might be difficult to categorize neatly; however, the categories are of some help in understanding the behavior of particular firms under particular stimuli.

Guise, whom we quoted earlier, also makes four interesting points which he labeled as survival essentials for a time-sharing firm:[1]

1. research and development in software predicated upon a belief that the real future lies in the sale of the system software as opposed to raw computer power,
2. a strong marketing force as opposed to reliance upon the customer's seeking and discovering your "better mousetrap,"
3. a nationwide capability since most sales are to divisions of large firms rather than to small businesses, and
4. the provision of a reliable service aimed at minimizing system "crashes" or periods of inaccessibility.

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[1]Huggins, "Guise Reviews Buying Habits," p. 49.

These four points may very well reflect the specific needs of Com-Share in the pursuit of its particular goals; however, they probably are fairly representative of the approach that many time-sharing firms are taking toward the market as they mature.

In summary, the overall picture is quite similar to either Fellner's or Bain's models (see Sections 2.3 and 2.4) of a few large firms with a healthy fringe of smaller firms. Obviously, in this case, a "fringe" seller from a national standpoint could actually be the dominant firm in its regional market.

3.7 Buyer Concentration and Other Buyer Characteristics

There is no evidence of significant buyer concentration in the time-sharing market. In a particular local market, there might be a few quite large buyers (e.g., a few aerospace companies). However, even in large firms, time-sharing is often purchased on a departmental basis rather than through centralized purchasing. This occurs because varying departmental needs might dictate the use of different vendors. Also, there is evidence that the average billing per customer is declining as the industry expands. GE is reported to have had its number of customers almost double from 1968 to 1969, its revenues rise only by 30 per cent, and its average billing fall from about $2000 to

$1000.[1] All these are indicative of lessening buyer concentration.

Some information is available on other characteristics of the buyers of time-sharing. A survey conducted among the readers of DATAMATION, a wide circulation journal in the computer area, revealed the following information:

The five largest users of time-sharing by industry (in order of descending use)

Aerospace
Petroleum
Electronics
Education
Chemical

The five smallest, separately listed users of time-sharing (again in order of descending use)

Medical
Service Organizations
Finance
Food & Drink
Publishing

The most important usages of time-sharing (in descending order)

Mathematical Computations
Statistical Analyses
Programming
Debugging of Programs
Simulation
Operations Research[2]

Another finding was that FORTRAN and BASIC are the most important languages by far with FORTRAN slightly more

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[1]Lundell, "Why the Shakeout?," p. 54.

[2]Bohdan O. Szuprowicz, "The Time-Sharing Users: Who Are They?," DATAMATION, August, 1969, pp. 55-56.

popular.[1]

Based upon these findings we can conclude that the most important market to date for time-sharing has been among technically trained users. This is indicated both by the nature of the heavy-usage industries and by the nature of the most common applications. This conclusion is supported by some of Guise's estimates. In 1968, time-sharing usage was distributed as follows:

> 70 per cent of sales were in the scientific-engineering market, of which 80% were to large firms owning large in-house computers.
>
> 15 per cent of sales were to the academic community.
>
> 15 per cent of sales were to commercial type users.[2]

In early 1970, Guise still reported that 75 per cent of the usage was the compilation or execution of user programs while only 25 per cent was the execution of system-provided applications programs. He further stated that he expected these percentages to eventually shift to 40-60 in the other direction.[3]

This industry has achieved its phenomenal growth by concentrating on the technical market which was the easiest market to penetrate. This market penetration was easier in

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[1] Ibid., p. 57.

[2] Robert F. Guise, Jr., "The '69 Time-Sharing Gold Rush," DATAMATION, August, 1969, p. 38.

[3] Huggins, "Guise Reviews Buying Habits," p. 49.

the technical or scientific market for several reasons: these customers have relatively heavy computing needs, they have often been given mediocre service by in-house computing centers, and they are often familiar with at least the rudiments of computer programming. However, if the time-sharing industry is to sustain high growth rates, effective ways of penetrating the less technical markets must be developed. One factor that might help is the development of time-sharing terminals which are both cheaper and more efficient for the non-scientific user to use.

3.8 Product Differentiation in the Time-Sharing Market

As we saw in Chapter II, the degree of product differentiation existing in a market has significant impact upon the degree of competition existing there. There is some product differentiation present in virtually every market--the question always is, to what degree? Let us examine the potential sources of product differentiation in this market:

1. The basic product of this industry is computing power and this is essentially a homogeneous product. In other words, it is possible to get the same output from any system even though the costs of doing it may vary widely for a given task. Each vendor, of course, does attempt to

stress that his system possesses invaluable features which makes it better than competitive systems. Nevertheless, these are frills--not the basic product.

2. Even in industrial markets, advertising can create or support significant product differentiation; however, in this market advertising does not seem to be a potent force. One reason perhaps for the quite limited use of advertising could be the rather heterogeneous nature of the buyers. There really is no single publication or group of publications which can reach a high number of prime sales prospects per dollar spent. Also, if a firm is regional or less in geographic scope, this limits its potential advertising media even more. At this time, the limited advertising that is used generally aims at promoting the concept of time-sharing more than advantages of the particular firm (i.e., primary rather than secondary advertising).

3. A seemingly more significant factor is the general image of the company, especially if it is affiliated with a computer manufacturer. GE, the leader in time-sharing sales, quite effectively uses the point that there must be a

reason why they are number one. This is the same argument which IBM has used very successfully for years in the general computer area. Especially for the beginning buyer of time-sharing, this affiliation with a known "name" might be very important.

4. Every time-sharing company has a library of programs which it stresses as being of fantastic usefulness to the user. With some exceptions, the value of the current library offerings of most firms is quite doubtful. Here, the larger firms do seem to have some edge, probably because both the developmental and documentation costs for library programs can be spread over a far wider base.[1]

5. Costs of changing vendors or contractual obligations can both provide some barrier to rapid buyer movement in the market. The contract

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[1]The development of software--especially sophisticated software--is a skill often more akin to an art than a science. Consequently, the input of more dollars does not always produce either a higher quality product or the same product in less time. The economies of scale are probably most predictable for the documentation aspect and for simple developmental tasks. Edwin Mansfield discusses the economies of scale in the general R & D area in The Economics of Technological Change, (New York: Norton, 1968), chap. 3. Many of his comments on development expenditures in general appear to be relevant to software development also.

factor is not too important since most contracts require at most a thirty day notice of cancellation. The costs of switching from one system to another can be significant, however, for the unwary user. Depending to the degree to which a user has availed himself of the peculiarities of his current system, he may find a substantial reprogramming effort involved in converting from one system to another.

A standard industry strategy seems to be to try to offer a superset of your competitors' capabilities thereby minimizing his effort in converting to your system. Once he has converted, encourage him to use many of your extended capabilities, thereby discouraging his conversion from your system.[1] Obviously, any moderately intelligent customer falls for this ploy just once. From that point on, he avoids any system frills which might trap him.

6. A buyer's ignorance and inability to evaluate

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[1]Arthur M. Rosenberg, "The Brave New World of Time-Sharing Operating Systems," DATAMATION, August, 1969, p. 47. This is an extension of Kenneth E. Boulding's "principle of minimum differentiation" which states, "make your product as like the existing products as you can without destroying the differences." Economic Analysis: Volume I - Microeconomics (4th. ed.; New York: Harper and Row, 1966), p. 484.

the many conflicting claims may make him susceptible to irrelevant sales claims until he pays his "tuition" in the market place. For example, a large system library might seem quite impressive to a buyer until he discovers that the majority of his needs require that he write his own programs.

Of all these factors, the last, buyer ignorance, may well be the most important at this stage of the industry's development. However, at any one time, there are only a limited number of lambs in the market place awaiting shearing.

One current shortcoming in the market is the lack of a good wide-circulation publication dedicated to time-sharing. Just as the lack of such a publication makes advertising unproductive, it also makes the exchange of information among users difficult. At present, word-of-mouth seems to be the most effective means of information flow.

To summarize, we see that the basic product, computing power, can be modified in various ways so as to appear somewhat different to buyers. However, despite their efforts, the vendors are not able to achieve the levels of differentiation which are common in consumer markets. In fact, there is only one actual barrier to buyer mobility--the danger of a buyer being locked in by conversion costs.[1] Even this usually affects only a limited number of buyers and is definitely a barrier of limited height.

3.9 The Condition of Entry to the Time-Sharing Market

Entry to this market must be relatively easy since there are certainly a lot of firms entering it. As indicated earlier, the number of firms has increased from none to over 200 in less than six years. Among the more important characteristics allowing this ease of entry are:

1. The capital investment necessary to launch a respectable operation is well within reason, especially in an area considered attractive (but no longer "red hot") to investors. A moderate scale operation could be started for one million dollars and a shoestring operation for ten to fifteen per cent of that.[2]
2. The technology, to date, has been rather easily obtainable. As the technology becomes more complex, this will no longer be so true.
3. The hardware and basic software are for sale to

[1]A substantial number of buyers (especially, the larger ones) employ on a regular basis the services of more than one vendor. Unfortunately, there are no good estimates of this percentage. See Szuprowicz, "The Time-Sharing Users," p. 58.

[2]For some examples, see "How the High-Fliers Take Off," Business Week, November 22, 1969, pp. 112-16.

anyone, and, in the case of slightly out-of-date but still adequate equipment, some real bargains are available.

4. There are good potential profits for the successful company and fabulous potential profits to the individuals who found such a company. Many companies, however, founded by brilliant technical people, undergo some harrowing experiences before learning some of the truths of the business world.[1]

5. The industry is growing at a very rapid rate and is currently able to absorb an expanding number of firms.

At present, this is a market in which sellers are entering (and leaving) continually. Such ease of entry may not continue indefinitely; as the accumulation of proprietary technology by existing firms continues, new firms may find it increasingly (and at some point prohibitively) expensive to match the then-existing level of technology.

There is no doubt that the greatest source of competitive pressure in this market is the ease of entry by new firms. There is also no doubt that many of these firms have no business entering the market and will not stay in it long.

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[1] *Ibid.*, pp. 112-16

3.10 Other Elements of Market Structure

In Section 2.7, we saw that there are several other elements of market structure whose presence tend to increase competitive behavior in a market. If we have (1) a high growth rate of market demand, (2) a high price elasticity of market demand, or (3) a high ratio of fixed to variable costs in the short run, we tend to have increased price competition. In the time-sharing market, the first and third of these factors are definitely present and the second may well be.

We earlier discussed the rate of industry growth and the general industry cost pattern.[1] The question of market price elasticity is not quite so clearcut. To the author's knowledge, no measurements of price elasticity have been attempted in this market. However, we can examine the characteristics of the product and draw some approximate conclusions.

As we saw earlier, there are close substitutes readily available for time-sharing when it is purchased for use in many applications. Also, even when the unique capabilities of time-sharing are needed (i.e., rapid turnaround and interaction), there are still substitutes avail-

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[1]Actually, in Section 1.4, we discussed production costs rather than total costs; however, development and distribution costs would have quite high fixed portions also.

able in another sense--many purchasers are large enough to set up their own systems.[1] On certain of its accounts, a time-sharing firm may encounter greater competition from the computer manufacturers than from other time-sharing vendors. The time-sharing area may well become one of the most important areas of make-or-buy analysis for many firms.

The existence of (1) products which are close substitutes, and (2) alternate means of obtaining the same product should cause a fairly elastic demand for computer time-sharing. Therefore, all three of the elements mentioned earlier seem to be present to encourage vigorous competition among time-sharing vendors.

3.11 The Hypothesis

We have examined the market structure rather thoroughly in accord with the theory contained in Chapter II. Although most market variables cannot be precisely quantified, we are still able to estimate approximate levels and make inferences based upon our approximations.

We are definitely faced with analyzing a differentiated oligopoly; the products are not perfectly homogeneous, nor is the number of vendors in any local market so large that the presence and actions of individual

[1]Another alternative would be for buyers to form co-operatives if the vendors collectively exerted monopoly power and raised prices enough so that it was economically attractive.

vendors are unnoticed by their competitors. However, market entry has been quite easy to date, providing regional competitors for the large nationwide marketers. Product differentiation, while present, is not evident to the extent it is in consumer markets; there is, for example, no intensive advertising of "brand names." Also, the relative "youth" of the industry (along with the horde of entrants) has probably deterred the growth of stable relationships among the competitors (e.g., collusion or price leadership). Games like this are difficult to play when the players keep changing.

This market does not appear to correspond to any existing "model" because of its unusual geographic structure. While the few large firms could easily establish quasi-agreements among themselves on the national level, they compete for sales at the regional level both among themselves and with the regional firms. Since all firms maintain a single price in all their markets, the establishment of quasi-agreements appears to be a fantastically complex concept in this case. This is so even without taking the continual change in "players" into account. One could probably argue with some merit that the regional vendors might even rely upon--rather than simply acquiesce to--leadership by the large firms. This could be especially true if the regional marketers had more faith in their technical skills than in their business skills. That is, they may be relying upon the large firms to establish reasonable price levels, policies, etc., more out of ignorance than out of any fear of economic reprisal by the larger firms. In fact, a small regional firm might almost be accused of corporate paranoia if it feared that it would cause a change in GE's national price level. To the author, it seems reasonable to suppose that the regional firms tend to compete rather vigorously just under the "umbrella" of the national price level established by the major firms.

A major factor allowing this study to be made is that we are not dealing with a consumer product. There are three factors which make computer time-sharing more suitable for analysis than a consumer product (e.g., the services of a physician).

1. Time-sharing is sold at present to firms rather than to individuals. Hopefully, the prime consideration of these firms in vendor selection is getting the "most" for their money. This is in contrast to the consumer market where highly subjective considerations (e.g., status) make analysis very difficult.
2. Much of the usage of time-sharing to date has been by scientific or semi-scientific users; these are people who should be quite capable of

some basic quantitative comparisons.

3. This "product" does lend itself to testing, even though thorough testing is not easy or simple.

None of these three factors would be similarly present in the case of most consumer products. We are saying in essence that the buyers in this market should be both motivated and able to "strip away" meaningless product differentiation and to evaluate the product on the basis of its actual productive capability.

Our basic approach in this study will be to assume that we do have a market structure in which price competition exists, hypothesize equality of effective prices between vendors, and test this hypothesis. However, rather than stopping there if the hypothesis is not accepted, we will relax our assumptions of no product differentiation and re-examine our results.

Our first problem then is the determination of whether the differences in nominal prices actually reflect the differences in vendor performance potentials so that the actual costs to the buyer of performing similar tasks are equalized from system to system. If the actual costs (and, therefore, effective prices) are fairly equal, then evidently this is a fairly competitive market with little product differentiation present.[1] The null hypothesis can then be stated as follows:

> Time-sharing services are sold to an economically motivated and analytically sophisticated group of customers in a market which is apparently fairly competitive; therefore, the apparent disparities present in the nominal prices are equalized by differences in performance such that effective prices are equalized.

The alternative hypothesis then becomes:

> Conditions in the market place are such that distinct product differentiation can and does exist as manifested by significantly different effective prices being charged to accomplish the same tasks with a similar level of performance.

3.12 Summary

Computer time-sharing is a young, technical industry which is experiencing tremendous growth. The current industry pricing structure is complex and definitely nominal. The question is whether the buyers are sophisticated enough and the competition is intense enough to have forced effec-

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[1]A finding of equal costs could of course support other conclusions such as price fixing or price leadership; however, neither of these seem as reasonable in light of the newness of the industry and the ease of entry, as mentioned earlier. The question of market conduct will only be answered with successive studies over time.

tive prices to equality.

CHAPTER IV

THE STRUCTURE OF INDUSTRY PRICES

4.1 Introduction

As our first step in developing a methodology for converting nominal prices to effective prices, we will carefully examine the structure of nominal prices in the time-sharing industry. This step is necessary to provide a framework for the subsequent steps of sampling and statistical analysis.

In order to align our terminology with that commonly used in the industry, we will refer to the vendor prices as "charges" but will use the term "costs" to cover any other buyer costs (implicit or opportunity).

First, we must identify the various component charges (and other costs) and, then, incorporate the relevant ones into our analysis. These cost components can be separated into four main areas:

1. running charges
2. storage charges
3. labor costs
4. other costs or charges

We will start by examining these four areas in turn.

4.2 Running Charges

A simple and widely used structure for time-sharing charges is made up of just three elements: connect charges, central processing unit (CPU) charges, and storage charges. Storage charges will be examined later. The charges for doing something on the system (i.e., performing some task) are based on just the first two charge elements. Connect charges are the charges for the time the customer is connected by telephone to the system. These charges are normally expressed on a per hour basis and are often in the $5 to $15 per hour range. CPU charges are the charges for the amount of central processing unit time used by the customer. CPU charges are usually quoted on either a per second or a per minute basis and are often in the $.03 to $.20 per second range. The ranges in both charges are a function of numerous factors such as:

1. the raw speed of the computer used,
2. the efficiency of the software used,
3. the number of users the system can (or is allowed to) simultaneously service, and
4. the type of customer which the vendor may wish to encourage

This list is not exhaustive; however, it does give some idea of the complex factors which vendors must consider in setting prices.

Some vendors do not consider this basic charge structure to be adequate for their needs; therefore they modify it in various ways. However, the one factor that is nearly always present is the connect charge. Unless a significant connect charge is imposed, a customer has no economic incentive to disconnect from the system and allow access by other customers. To achieve reasonable utilization levels, the typical time-sharing service must have many more customers than it has access "ports" to its system, since many customers will only wish to use the system a portion of the day. Therefore, the connect charge performs the function of rationing access ports among the users. Although this connect charge is almost always present, it can vary considerably from firm to firm.

One method of altering the charge structure is to retain the concept of only two running charges but to make one of the "time" units--normally the CPU time--more complex. The basic processing (CPU) time for a task can be modified to reflect factors such as the amount of memory required by the task, the number of times the task accesses auxiliary memory, the use of proprietary system programs, or any other specialized factor which the vendor may wish to include. The name of such an adjusted time unit may or may not reflect what it includes. For the purposes of this model, we can simply accept any such unit as the processing time. How the accounting system arrives at it, to a large extent, is immaterial.[1]

An additional (third) charge which is occasionally encountered is an input/output (I/O) charge which can refer to either the input/output transmitted back and forth over the telephone line or to the input/output between the central processing unit and the auxiliary memory units. Again, regardless of the source of the charge, it simply represents an additional charge element for running a program.

The total charge for running a program is the sum of connect charge, the processor charge, and the input/output charge. On a given system, any (but not all) of the three costs may be omitted from the system charge structure.

We can now describe the concept of running charges in symbolic terms. For our purposes, we will describe these charges in terms of performing some particular task upon some particular system where the term "system" is defined as a given computer language upon a given vendor's time-shared computer. Therefore, a single vendor could conceivably have many systems, but the normal number will be two as we will see later. To aid us in identifying the particular charge elements, we will use the following system of subscripts.

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[1]Conversations with many users have led the author to the general conclusion that customers are concerned with the "what" and not the "how" or "why" of system charges.

Let:

α denote an alphabetic variable (i.e., A, B, C, . . . Z) used to identify a particular system,

i denote the identifying number of some particular task, and

j denote the replication number[1] (e.g., the second performance of task i on system α).

Using these subscripts, we can now define the individual elements of the running charges.

Let:

$RC_{\alpha ij}$ denote the total running charges for system α to perform task i for the j^{th} time,

$CC_{\alpha ij}$ denote the connect charge for system α to perform task i for the j^{th} time,

$PC_{\alpha ij}$ denote the central processor (CPU) charge for system α to perform task i for the j^{th} time, and

$IC_{\alpha ij}$ denote the input/output charge for system α to perform task i for the j^{th} time.

Therefore, to determine the total running charges for performing the i^{th} task for the j^{th} time on system α, we have:

$$RC_{\alpha ij} = CC_{\alpha ij} + PC_{\alpha ij} + IC_{\alpha ij}$$

However, there are other elements of cost to using a system besides running costs; therefore, we will now incorporate

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[1]The term "replication" is normally used to describe repeated performances of the same experiment to obtain average values.

these additional costs.

4.3 Storage Charges

Storage charges are imposed for the storage of the user's library which is comprised of programs and/or data files. This library resides in the system's auxiliary memory units which are usually magnetic disc or drum units. This ability to store information frees the user from having to enter his program or data every time he wishes to use the system. Instead, the user can easily recall previously stored information from his library. The choice of what to store and how long to store it lies with the user.

The charges for storage can be somewhat confusing since there is considerable variety in the way they are quoted. For example, charges can be based upon the maximum storage space used during a month or upon the average of a number of "readings" made during the month. Unless the size of the user's library is virtually fixed, the average basis is certainly better for him (assuming the same rate per amount of space). Another possible source of confusion is the concept of the "unit" of storage. This unit is the smallest block of library space which the system can allocate. If any part of a unit is used, the entire unit is automatically used. Suppose, therefore, that we have two time-sharing services, A and B, which both quote a <u>nominal</u> storage rate of $1.00 per 1000 characters per month.

However, the true rate for A might be \$.80 per 800 characters per month (i.e., unit size is 800 characters), while the true rate for B might be \$.60 per 600 characters per month (i.e., unit size is 600 characters). To store a 2500 character program for a month would cost \$3.20 (4 units at \$.80 per unit) on system A and \$3.00 (5 units at \$.60 per unit) on system B. The general rule is that, for any given nominal rate per character, a smaller unit size is better for the customer.

Comparisons of storage costs can become quite complex if services are compared which have different costs per character, different unit sizes, and differing methods of recording usage (average vs. maximum). One other complicating factor could be the inclusion of some "free" storage in the base contract. We will handle these details in Chapter V; for the moment, we will assume that we can determine the storage charge associated with performing a particular task.

Following the subscripting conventions described earlier, we can now incorporate storage costs into our model.

Let:

$SC_{\alpha ij}$ denote the storage costs associated with performing task i for the j^{th} time on system α.

We have now examined all of the major cost elements;

however, we should make some provision for possible miscellaneous costs.

4.4 Miscellaneous Charges

There are many types of minor charges which various systems might have. For example, there can be charges for extra account numbers, changing passwords,[1] or special billing arrangements. These charges are rather unimportant and, in fact, are normally instituted by the vendor solely to discourage abuse. There are, however, two other types of charges which we should examine. One is the initiation charge and the other is the monthly minimum charge.

The service initiation charge is a one-time charge and is generally $100 or less.[2] Many firms do not have this charge at all. It is also quite probable that most vendors having the charge would not lose a prospective good account (i.e., substantial billings) by insisting upon this charge. Regardless of this, it is relatively unimportant in the sense that a $100 initiation charge would have declined to an $8.33 average monthly cost after only one year.

A monthly minimum charge is imposed by a portion of

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[1]A password, as its name implies, is the user's key to accessing the time-sharing system. If his password becomes too widely known (or if he forgets it), a new password must be established.

[2]"General-Purpose Time-Sharing Survey."

the firms and is typically $100 per month.[1] For many users, this cost is not too relevant since it is a minimum and not a "cover charge." As previously mentioned (Sec. 1.4), GE's average billing after a sharp decline is still about $1000 per month. Certainly, the presence of a minimum could affect the <u>very</u> light user; however, most time-sharing services seem increasingly uninterested in obtaining or maintaining very low volume accounts.[2] Many of the costs of selling and servicing a customer are similar if he buys $50 or $1000 worth of time-sharing per month. Again, for the purposes of this study we will ignore the minimum, assuming that it is not conduct-determining for the typical user.

Despite the above downgrading of the importance of various miscellaneous charges to this study, we should still have a provision for incorporating them into our analysis just in case some unforeseen factors might arise during the course of the experiment. Therefore, we will

Let:

$MC_{\alpha ij}$ denote the miscellaneous charges associated with performing task i for the j^{th} time on system α.

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[1] *Ibid.*

[2] Drake Lundell, "GE's Information Systems De-Emphasizes Time-Sharing Services for Small User," *COMPUTER-WORLD*, April 22, 1970, p. 1.

4.5 Total System Charges

We have now examined all the sources of vendor charges for time-sharing services. The total charges by the vendor are the sum of running charges, storage charges, and miscellaneous charges. Or, if we

Let:

$DC_{\alpha ij}$ denote the total charges by system α to perform task i for the j^{th} time,

then

$$DC_{\alpha ij} = RC_{\alpha ij} + SC_{\alpha ij} + MC_{\alpha ij}$$

4.6 Labor Costs

We must now examine those costs imposed upon the time-sharing buyer which are in addition to the charges actually billed by the system. We are however only concerned with charges which would vary according to the system used. For example, a time-sharing buyer must pay for a terminal regardless of the system he uses; however, we would be concerned with this terminal cost only if it varied from system to system.

The prime example of an "additional" cost which varies from system to system is the labor cost of the system user. This user is the person who actually sits at the terminal during the time the computer is being used. His labor costs are significant in this study to the extent that using different time-sharing systems causes labor cost differentials to exist. The minimum labor cost to perform a task on time-sharing must be accepted as soon as the decision is made to use time-sharing. What must be considered in system comparisons is the cost of the extra labor required by any system other than the "fastest" one. In other words, this is a surcharge applied to differential connect times.

It is conceivable in some cases that a longer wait (especially if its duration is predictable) should actually bear a lesser cost than a shorter wait; an employee might productively use a longer wait but not a shorter one. As we will see later, the design of the experiment rather eliminates this assumption from our analysis.

The labor cost for waiting is based upon the excess average connect time of a particular system compared to the lowest average connect time for any system. That is, if we

Let:

$LC_{\alpha ij}$ denote the labor cost of the extra waiting imposed by system α in performing task i for the j^{th} time,

then

$$LC_{\alpha ij} = WC(t_{\alpha ij} - \text{minimum } t_{ij})$$

where:

WC denotes the labor cost/unit of time spent waiting (this is constant for any given user),

$t_{\alpha ij}$ denotes the connect time necessary to perform task i for the j^{th} time on system α,

and

minimum t_{ij} denotes the lowest connect time necessary to perform task i for the j^{th} time on any system.

Note that while the previous charges we have discussed have been explicit dollar charges by vendors, $LC_{\alpha ij}$ represents an opportunity cost which is hidden in the customer's total labor costs. This, of course, does not make $LC_{\alpha ij}$ any less real, only less apparent.

There may be some other minor implicit costs which vary from system to system but the author has not been able to identify them as such. Also, any other such costs would probably be highly subjective as opposed to the labor costs which we can incorporate into our analysis with reasonable ease and accuracy.

4.7 Total Buyer Costs

We can now incorporate all the elements into our structure and obtain an expression for the total costs to the buyer, remembering however that we only have "total" costs in our limited sense (i.e., total system charges plus any additional costs that vary with the system used). If we

Let:

$TC_{\alpha ij}$ denote the total cost of performing task i for the j^{th} time on system α, then

$$TC_{\alpha ij} = DC_{\alpha ij} + WC_{\alpha ij}$$

where $DC_{\alpha ij}$ and $WC_{\alpha ij}$ refer to the system charges and labor costs as previously defined.

If we perform each of the i tasks m times,

then

$$\overline{TC}_{\alpha i} = \frac{1}{m} \sum_{j=1}^{m} TC_{\alpha ij}$$

where

$\overline{TC}_{\alpha i}$ denotes the average cost of performing task i on system α for m times.

Furthermore, if we perform a total set of n tasks,

then

$$\overline{TC}_{\alpha} = \frac{1}{n} \sum_{i=1}^{n} TC_{\alpha i}$$

where

$\overline{TC}_{\alpha}$ denotes the average cost of performing all n tasks on system α.

4.8 The Null Hypothesis Restated

The null hypothesis (Section 3.11), stated in terms of our economic model, is that effective prices in the market place are equal despite the wide variations in nominal prices. To test this hypothesis, we must compare the costs to the user of performing a typical set of tasks on various systems. Therefore the null hypothesis, stated in terms of our analysis, is that the average total cost of performing a set of n preselected, typical time-sharing tasks is equal from system to system. That is,

$$\overline{TC}_A = \overline{TC}_B = \overline{TC}_C = \ldots = \overline{TC}_Z$$

This restatement provides the link between the null hypothesis of our study and the empirical analysis of Chapters V and VI.

4.9 Summary

We have now examined the various cost components of using a particular time-sharing system, including both the vendor charges and the opportunity costs of waiting. We have also integrated these various buyer costs into a single framework. The hypothesis of cost equality has also been restated in terms of this framework. The next step is to define the parameters and procedures of a suitable experiment so that the hypothesis can be empirically tested.

CHAPTER V

THE DESIGN OF THE EXPERIMENT

5.1 Introduction

Having examined the pricing structure of the time-sharing industry, we must now examine the design of the experiment which was used to gather data for the statistical analyses. The design of any experiment usually involves a compromise between theoretical and pragmatic[1] considerations, and this experiment was no exception as we will see. First, let us enumerate the general steps in the design problem and, then, examine each step in detail.

The first step was the development of a reasonable unit of time-sharing product (i.e., a set of standardized tasks) as viewed from the standpoint of a "typical" user. After the concept of the product was developed, uniform procedures were developed for measuring the relative costs of obtaining the unit of product on various systems. Once these testing procedures were developed, the systems to be tested were selected, and, finally, the necessary system measurements were made. We will now examine each of these

[1]The pragmatic considerations in experimental design are usually problems of economics; however, there can be physical problems such as constraints on time or subject availability.

steps in turn.

5.2 A Unit of the Product

In the time-sharing industry, it is impossible to construct a definition (i.e., unit of measurement) of the product which is both simple and useful. The vendors are selling time-units of computing power (which vary in quality from vendor to vendor), while the buyers are purchasing computer power relative to their needs (i.e., problem solving ability). There are, therefore, two main sources of confusion facing a buyer. First, there is the problem of framing *his* needs in terms of *any* vendor's product. Even if this were possible (without testing), the problem of converting the units of one vendor to those of other vendors would still remain. The confusion is accentuated by several additional factors. First, system performance is a function of both the hardware and the software.[1] Therefore, two services using essentially identical hardware but different proprietary software could have totally different performance patterns relative to the needs of a particular buyer. Secondly, there are a number of

[1]The terms "hardware" and "software" are widely used to denote the difference between the physical machine and the programs and other support necessary to use the machine. In addition, software is termed "proprietary" if it is protected by the developing company rather than being put in the public domain. The general trend is toward more proprietary software.

subjective factors which may or may not be important to a particular buyer. These include library programs, educational offerings, systems engineering support, and system reliability.[1]

The only technique currently available to combat this complex decision problem is the actual measurement of performance needs and characteristics by trials performed on the systems of various vendors. In the time-sharing industry, these performance (and cost) tests are known as "benchmark" tests. The buyer is hoping, of course, that his benchmark tests will locate the time-sharing system which has the lowest effective prices.

When a particular buyer performs benchmark tests, the tasks which he chooses as his benchmarks are (hopefully) a representative cross section of his time-sharing needs. In this study, the problem is one of wider scope. Here, the tasks must be representative of the general needs of most time-sharing purchasers, not just the particular needs of a single user.

Fortunately, this task is not quite as difficult as it first seems. Time-sharing users who might be quite different in some ways can be quite similar in their demands

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[1]For a fairly comprehensive checklist of these subjective considerations, see Alan G. Hammersmith, "Selecting a Vendor of Time-Shared Computer Services," COMPUTERS and AUTOMATION, October, 1968, p. 21.

upon the capabilities of a time-sharing system (even if their tasks also seem different). For example, an engineer running his stress analysis program and a psychologist running his factor analysis program could use very similar blends of time-sharing system resources. In other words, both might use about the same relative amounts of connect time, CPU time, and storage on their seemingly dissimilar tasks. In contrast, for example, someone using a system for accounting work might use large amounts of connect time and large amounts of storage but almost no CPU time. Therefore, a set of tasks had to be chosen which would typify the <u>blends of demands</u> which typical users impose upon their time-sharing system.

The choice of the set of representative tasks for this study was rather subjective; various objective factors were considered, but there are (and can be) <u>no</u> quantitative measures which can neatly choose a good package of tasks. For this study, nine separate tasks were chosen. No particular significance is attached to the number nine; this merely happened to be the number of tasks which evolved as being necessary to encompass the desired tests of system capabilities based upon the author's experience.

Tasks 1 through 6 are programs written especially for this study to test isolated system characteristics (e.g., compute only vs. compute interrupted by output).

Tasks 7 through 9 are programs which were adapted for this study, and they represent typical productive programs for numerous types of users. These programs and their individual characteristics are as follows:[1]

Task #1 - 100,000 integer additions with no input and little output. Integer addition was chosen because it is commonly used as a test of raw machine speed.

Task #2 - the same 100,000 additions as Task #1; however, once every 2,000 additions (i.e., 50 times) it prints the following: "THIS IS LINE NUMBER n." There is no input.

Task #3 - the same 50 print lines as Task #2 but without the 100,000 additions.

Task #4 - 50 requests to the user to input a number at the terminal. After each input, the system prints, "THIS IS LINE NUMBER n," and requests another input.[2]

Task #5 - the same 50 requests to the user as in Task #4; however, the lines of output are omitted.

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[1]See Appendix I for listing of the BASIC versions of all nine programs.

[2]Tasks #4, #5, #7, and #8 are interactive with the person executing them. These introduce special procedural problems which are discussed in Section 5.3

Task #6 - a test of the speed of the system in handling the mathematical functions in the language. The program evaluates 10,000 times the following expression:[1]

```
LET X = ABS(SIN(INT(SQR(100*RND(0)))))
```

This expression (which has no practical application) finds the absolute value of the sine of the integer part of the square root of decimal random numbers between 0 and 100. Every 1,000th result is printed.

Task #7 - a Monte Carlo simulation of a single queue-single station queuing model. The program performs 1,000 simulations and prints, at intervals of 100 simulations, the current and cumulative averages for number of arrivals, waiting time, queue length, service time, and facility utilization. The input of the required parameters is requested from the user, item by item, through the terminal. These parameters include the probability

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[1]This is the BASIC language version. The FORTRAN version performs the same sequence of operations but is slightly different in appearance because of the syntactical differences in the two languages.

distributions for demand and lead time, the length of the simulation period, and the desired number of simulations.[1]

Task #8 - a linear programming program which leads the user interactively through the constraint and objective function input. The program produces the maximized or minimized value of the objective function and, also, the identity and quantity of each of the variables included in the final solution. The user is then able to modify any selected input value and obtain new results. In this study, a sample problem of fifteen variables and eight constraints was solved with one modified version being solved also.

Task #9 - a regression analysis which can handle up to forty observations of each of two to five variables (one dependent and one to four independent). This data is entered from a data file[2] in the system auxiliary

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[1]Samples of the input and output for Tasks #7, #8, and #9 are shown in Appendix II.

[2]The data is placed into this data file by another program which is not part of the test package.

storage rather than through the terminal. The program then performs simple regressions, tests for autocorrelation, performs a multiple regression, computes simple parabolic regressions, and, finally, examines the simple linear intercorrelations. In addition to merely computing the above statistics, the program generates numerous standard deviations and error estimates for these statistics. For this study, a data set consisting of twenty observations of each of four variables was used.

Any of the latter three programs could be used in either exact or modified form by many users of time-sharing. In fact, the linear programming and regression programs are similar to programs often found in system libraries.

The general characteristics of the selected nine tasks described above are as follows:

1. The tasks were weighted toward both the types of users and usages outlined in Section 3.7 (i.e., scientific type tasks).
2. Every task can be performed, without significant re-programming, on most current time-sharing systems. This restriction was necessary to

insure that the tasks remained constant from system to system. The only programming changes allowed were those necessary to adapt to trivial differences in syntactical requirements from system to system.[1] In other words, no differences were allowed in either the number or type of instructions included in the programs.

3. The tasks are all written in both of the two most common time-sharing computer languages, BASIC and FORTRAN.

4. The three tasks involving relatively long programs were all adapted from programs originally written by others in order that a varied assortment of personal programming techniques might be included.

5. All tasks were defined to include compilation and execution although most systems allow the storage of executable code.[2] The inclusion of

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[1]It was a quite time-consuming task to establish a core of capabilities which were common to all systems. No program could be allowed to reflect any peculiarities of the systems upon which it was originally developed.

[2]No computer, time-shared or otherwise, can directly execute a program written in anything but numeric codes. To execute a program written in FORTRAN or BASIC, there must be an intermediate step to translate the program to executable code. If a system allows the storage of executable code, the intermediate step can be omitted on subsequent executions.

compilation for each execution was designed to reflect the importance of programming and debugging as a time-sharing usage (see Section 3.7).

6. The running times of the tasks were balanced (i.e., set at general levels) such that no ridiculous extremes should be encountered (e.g., running in either no time or several hours) from system to system.

7. Four of the nine tasks were made moderately-to-highly interactive. This reflects the fact that interactivity between the user and the computer is one of the basic advantages of time-sharing (see Section 1.3).

In the final analysis, the exact composition of the tasks was based upon the author's extensive personal experiences with time-sharing and upon his countless conversations with both users and vendors.[1]

This package of nine programs was designed to impose a broad range of physical demands upon a system. Breadth in testing is necessary to insure that no system is able to appear attractive in a few tested areas at the expense of other areas which might not be tested. Although

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[1]The development, conversion, preliminary timing, and balancing of the nine test programs required an estimated $3,000 worth of computer time.

the test package is broad in a total sense, the emphasis was still placed upon the needs of scientific type users. Of the nine tasks, six are quite representative of the types of programs commonly performed by the scientific user.

5.3 Establishing the Testing Procedures

Once the mix of test programs was selected, the next step was the development of testing procedures. We will examine these procedures and the reasons, both theoretical and pragmatic, behind each.

One constraining factor in all phases of the experimental design was that this was not a funded study; therefore, the computer time necessary to conduct these experiments had to be obtained without charge from the firms in the industry. Access to any system was normally available for a few days only, and there was an implicit obligation not to abuse the privilege of access.

One major decision was that each of the nine tasks should be performed three times (i.e., three replications of the experiment). The number three represents a definite compromise. More replications should produce better estimates of the average charges but would also increase the total cost of testing (i.e., the imposition upon the vendors).[1] Preliminary testing indicated that three replications was a reasonable compromise between accuracy and cost. For example, one such test showed that the sample means of samples of size one averaged approximately 5.6% error relative to the "true" mean; samples of size two, 3.7% error; samples of size three, 2.8% error; and samples of size four, 2.3% error. The "true" mean was determined in this case by running the same program ten times. Although the marginal accuracy declined rapidly as shown above, each additional replication still increased the imposition upon the vendor by approximately $30 to $40 worth of computer time. Therefore, the author decided that three replications were necessary and justifiable but that more than three were not of sufficient benefit to the study to warrant the marginal imposition.

Once the decision of three replications was made, some schedule had to be devised for performing the 27 tasks (i.e., three replications of nine tasks). In line with common design practice, it was decided to randomize the order of performance on each system. Therefore, random testing orders were generated using a time-shared computer program. However, some additional decisions were also

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[1]Basic sampling theory tells us that increases in sample size lead to less than linear increases in accuracy; however, changes in sample size do tend to increase testing costs linearly as they would in this case.

necessary to provide input to this scheduling program.

Since there is a definite pattern of time-sharing usage over the daytime period, the times for testing had to be considered. A system's performance (and, therefore, its charges) can vary considerably according to the load upon the system. Connect time is especially sensitive to system load on certain types of tasks. A typical usage pattern,[1] as shown in Figure 5.1, has peaks at approximately 11 a.m. and 3 p.m. daily. To create time schedules, two things were necessary: the starting time which was set at 9 a.m., and estimates of testing time for each program. Based upon some trials, the testing times were set at five minutes for tasks #1, #2, #3, #5, and #6; ten minutes for task #4; and twenty minutes for tasks #7, #8, and #9. These times were intended to be ample <u>in most cases</u> thereby allowing slack in the schedule to meet emergencies. Using these assumptions, testing schedules such as the example shown in Appendix III were generated for each system. Note that the determination of charges or costs was in no way based upon the amount of slack time. Slack time was included only so that <u>all</u> the tasks (which required differing times from system to system) would be performed in the same <u>total</u> time

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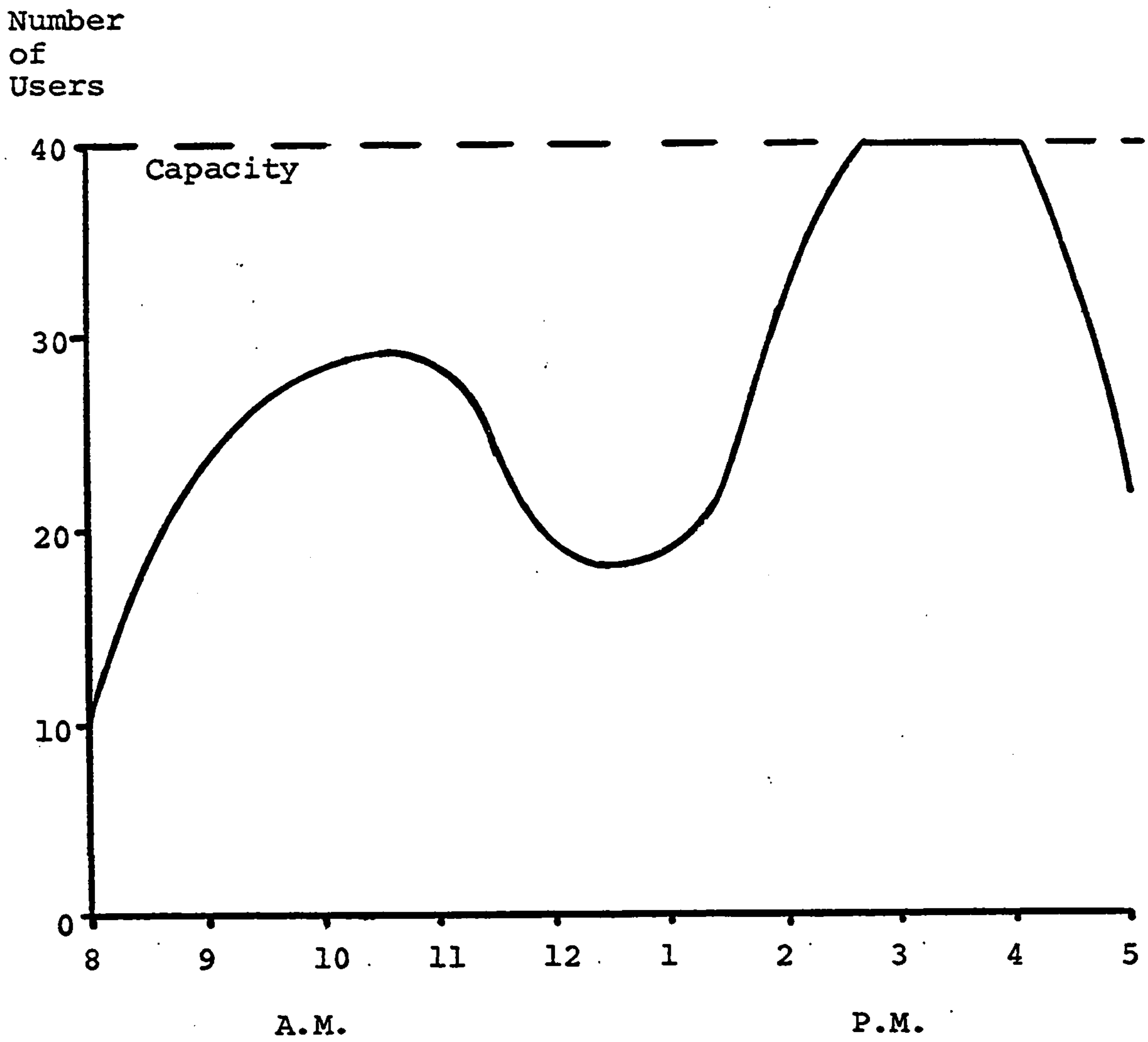
[1]These patterns are discussed by Szuprowicz in "Time-Sharing Users," p. 58. One interesting sidelight to these usage patterns is that they indicate that East-West expansion for a firm is preferable to North-South expansion, all other things equal.

FIGURE 5.1

TYPICAL TIME-SHARING USAGE PATTERN
BY TIME OF DAY



Source: Szuprowicz, "Time-Sharing Users," p. 58.

span.

During the testing, the procedure was to execute each task as scheduled if possible. If for any reasons the testing fell behind schedule, testing was continued without any delays (i.e., utilizing the scheduled slack time) until the normal schedule could be resumed. Very little trouble was encountered in keeping reasonably close to schedule because the original time allowances were sufficiently generous. Some deviations did occasionally occur usually because of either temporary system failure or a particular program exceeding its time allotment.

Some of the data was obtained from information provided by the system. Processing times, I/O times, and storage units were all obtained in this way. Connect times for the tasks were measured to the nearest second by stopwatch.

The measurement of connect times did pose a potential problem in one sense. On programs which are interactive, the problem arises of standardizing the user's reaction time to system requests for input. This time which the system spends waiting for the user to answer is often called "think" time as opposed to "response" time which is the time which the user spends waiting for the system.[1] To make valid comparisons between systems, think time must be kept relatively constant since variations in think time represent variations in the performance of the user, not the system. In this study, the same input data was always used for a task, and it was always entered as quickly as the system requested it. Any experimental error arising from this source was negligible.

In summary, each of the nine tasks was performed three times in randomized order on each system following similar schedules. Standardized testing procedures were used in an attempt to minimize or randomize the potential sources of experimental error such as times of day or varying think times. Any experimental error should represent actual deviations in system performance.

5.4 Selecting the Systems

The problem of system selection was really a problem of vendor selection. The six vendors selected each provided two systems since all of the vendors had both FORTRAN and BASIC systems which were acceptable.

Several factors were considered before selecting the six vendors actually tested:

1. The two dominant vendors in the time-sharing

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[1]Alan Lee Scherr presents these and other more technical concepts of interactive computing in An Analysis of Time-Shared Computer Systems, M.I.T. Press, 1967, pp. 4-9.

market, General Electric and the Service Bureau Corp., had to be included since they share over 50 per cent of the national market (see Section 3.6).

2. Each vendor had to be servicing the Cincinnati area; however, the vendor had to be at least regional in his total marketing scope. The Cincinnati service was necessary to provide accessibility; the regional service was necessary to provide something more meaningful than a study of just the Cincinnati market.

3. Each vendor had to be an apparent long-run entrant in the time-sharing market. This was determined by questioning representatives of each firm about the details of their firm's plans for the future. Such topics as personnel expansion, product improvement, equipment upgrading, etc., were covered. While the answers may vary from company to company, this type of questioning can reveal whether a firm does have reasonable plans for continuing as a viable market entrant. The aim of this questioning was to eliminate any firms which might be short-run profit maximizing in order to maximize the owners' gains through stock manipulation.

4. A varied cross-section of computer systems was desired in order to better typify the variety of systems used in the general market.

5. Each vendor had to have a typical product. Systems mainly oriented to test editing, accounting, or file management were eliminated. All vendors had to provide the medium to large computer capabilities necessary to handle meaningful scientific type programs.

Using these criteria, the vendors shown below were selected.

Vendor	Type of Computer	Location of Computer
Applied Computer Time-Sharing	GE-430	Cincinnati, Ohio
Cyphernetics	PDP-10	Ann Arbor, Michigan
Direct Access Computing	Burroughs B-5500	Southfield, Michigan
General Electric	GE-635	Cleveland, Ohio
Service Bureau Corp.	IBM S/360-50	Cleveland, Ohio
Structural Dynamics Research Corp.	GE-420	Louisville, Kentucky

Of these six firms, two, GE and SBC, are nationwide marketers. The other four market across varying areas of the Midwestern United States. Taken together, they should approximate most urban markets.

As discussed in Section 3.6, all firms do charge the same prices in all markets in which they elect to compete. This removes one hindrance to the general applicability of the results of this study. If the nominal prices of vendors changed from location to location, each small geographic area would have to be carefully studied to determine if the same relationships existed. Fortunately, standardized prices are the accepted pattern.

Once the firms were selected, the tests were conducted. Ideally, the elapsed time from start to end in such a study should be as close to zero as possible. Here, however, some very definite practical considerations extended the testing period for this study over a three month period (approximately the first quarter of 1970). Working alone and using vendor supplied time, the author felt fortunate in keeping the time span that short. The primary reason for concern with the time span is that the average system loads can change over time (up or down). Just as time or day affects performance and charges, so does the average system load with heavier loads leading to poorer performance and higher charges. Since there was no available method of shortening the total time span, any inaccuracies introduced through this factor must simply be accepted as inevitable.

5.5 Setting the Labor and Storage Parameters

Before the results of the tests could be statistically analyzed, two additional factors had to be determined. The first was the cost per hour of waiting time, defined as WC in Section 4.6. The second was the exact method of allocating storage charges program by program.

In line with the information that both the users and usage were primarily scientifically and/or technically oriented, the value of WC was set at $7.00 per hour. This value reflects a total annual employment cost of $14,000 to $15,000 per year, a reasonable value for technical personnel. This hourly rate is only applied to differential labor times as indicated in Section 4.6.

The storage problem was somewhat more difficult to handle for the reasons cited in Section 4.3. An additional complication arose because storage charges are imposed for storing a program over a unit of time (typically one month) while running charges are imposed for any one performance of a program (task). What portion of the monthly storage charge should then be allocated to any one performance of the program? The method decided upon was to assess each program 10 per cent of the charge for storing it for one month <u>if</u> the particular system computed storage charges on an average usage basis. The 10 per cent figure assumes that performing a given program ten times per month is a realistic figure for average usage. For systems which use the maximum rather than average storage concept, the storage charge was assessed at 12.5 per cent rather than 10 per cent. The 2.5 per cent difference was designed to impose a 25 per cent penalty on the non-averaging system. The 12.5 per cent value does not imply that the user on the maximum type system only uses his program eight times per month. It instead means that he typically tends to be charged for 25 per cent more units of storage a month than he would be if the system he was on used the averaging technique instead.

All of the values assigned are arbitrary in one sense; there is no data available to support them. On the other hand, the author has had extensive experience with both types of systems, and, based upon this experience, these seem to be reasonable estimates.

5.6 Summary

In this chapter, we have examined the actual methods used in conducting the tests for this study. Also, we have looked at the problems which gave rise to this particular methodology. We are now ready to examine the experimental data and the statistical analyses which were applied to them.

CHAPTER VI

THE STATISTICAL ANALYSIS

6.1 Introduction

In any study involving sampling, statistical testing is necessary to determine whether the deviations of the sample values from the hypothesized population values (or relationships) are too great to be reasonably attributable to chance. If chance cannot account for deviations of the observed magnitude, the statistical conclusion must be that the null hypothesis is not true. Since sampling normally involves testing only a portion of the population, there is always a chance of error--either rejecting a true hypothesis or accepting a false hypothesis. The choice of the most appropriate statistical test for the given sampling situation can minimize the probability of either type error.

After the sampling experiment described in Chapter V was conducted, the individual measurements had to be processed and assembled into groups and subgroups before any general statistical analyses could be chosen or performed. The choice of the proper statistical analyses was to some extent data dependent; that is, the data had to conform to the assumptions underlying the analyses. Therefore, some preliminary statistical analysis was performed upon our observed data to aid in determining which types of major analyses could be correctly applied to our data. Based upon these preliminary findings, the proper tests were selected and executed.

In this chapter, we will first examine the data sets which were formed. Next, we will review each of the potentially relevant statistical analyses. For those which proved acceptable and were used, we will review and interpret the results relevant to the original hypothesis.

6.2 Organization of the Data

Before even preliminary statistical analysis could begin, the data had to be compiled into its final form in accordance with the cost structure shown in Chapter IV. The running times and charges for each system are shown in Appendix IV. To these charges, the storage charges shown in Appendix V and the labor costs shown in Appendix VI were added to produce the total costs shown in Appendix VII. These total cost values were the input to the statistical analysis portion of the study.

In addition to the basic analysis of the total group of twelve systems, analyses of three additional groupings seemed potentially helpful. First, the twelve systems were divided into two subgroups: (1) the BASIC language systems, and (2) the FORTRAN language systems. Each vendor is represented once in each subgroup since all six vendors offered both languages. In case significant cost differences might exist between the two languages, this subgrouping would yield more homogeneous groups to study. The six BASIC language systems are A, C, D, G, I, and K; the six FORTRAN systems are B, E, F, H, J, and L.

The third additional group represents the six vendors. This group was created by assigning to each vendor the average cost of his BASIC and FORTRAN systems.[1] The vendors can be thought of as six other systems labeled U, V, W, X, Y, and Z. These six can then be tested against one another to give a better picture of a vendor's total package.

For many of the statistical tests, there were, therefore, four data sets as input. The null hypothesis was always that there was equality of average total cost.[2] The economic significance of accepting or rejecting the hypothesis varies of course according to the particular data set as we will see.

The average total costs of performing all the tasks on each system are shown in Table 6.1 along with the vendor

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[1]Averaging the two cost values gives FORTRAN and BASIC implicitly equal weights. While this may not reflect their exact importance, it should be close enough for our purposes.

[2]Some tests will require a somewhat different null hypothesis; however, equality is always the object of the search.

average total costs. The BASIC and FORTRAN systems taken together make up the twelve system group. Also, the BASIC and FORTRAN systems are correctly paired horizontally to make up their respective vendors (e.g., vendor V is selling BASIC system C and FORTRAN system F). The function of the statistical analyses will be to tell us whether the differences among these average system costs are statistically significant.

TABLE 6.1

AVERAGE TOTAL COSTS

BASIC		FORTRAN		Vendors	
System	Cost	System	Cost	System	Cost
A	$6.36	B	$3.99	U	$5.18
C	3.46	F	5.91	V	4.68
D	5.19	E	7.12	W	6.15
G	3.37	H	2.26	X	2.81
I	4.06	J	4.32	Y	4.19
K	4.01	L	2.39	Z	3.20
Average	$4.41		$4.33		$4.37

6.3 Analysis of Variance Test

This experiment was designed with the analysis of variance (ANOVA) test in mind. In ANOVA terms, the systems are one factor, the tasks are another factor, and the repetitions of the tasks are replications. This study provides a mixed model with the blocks as fixed factors and the treatments as random factors. For any cell (i.e., task-system combination), the contents would be as follows:

	System α
Task i	$TC_{\alpha i1}$
	$TC_{\alpha i2}$
	$TC_{\alpha i3}$
	$\overline{TC}_{\alpha i}$

Performing an ANOVA test would determine whether the system averages or $\overline{TC}_{\alpha}$ values are significantly different. However, to determine the applicability of the ANOVA test, we must first examine the underlying assumptions.

The ANOVA test is a parametric test meaning that the test is valid only in cases in which the data conforms to certain assumptions as to its distribution. These assumptions are as follows:

1. The effects of each factor and of any interactions between them must all be additive.
2. The experimental errors (i.e., variations between replications) must be independent.
3. The experimental errors must be normally distributed with equal variances.

All of the assumptions must be fulfilled although minor deviations may usually be ignored.[1] Generally, a violation of any one of them has a fairly predictable direction of effect but an uncertain magnitude of effect. The only assumption which is easily (and commonly) tested is the homogeneity of variances.

Two separate tests of homogeneity, Cochran's and Bartlett's, were conducted. These tests were applied to the data of all twelve systems and to the FORTRAN and BASIC subsets. Both tests are designed to discover hetero-schedasticity (the lack of homogeneity in variances) based upon samples drawn from normal populations. The Bartlett test is the more powerful (i.e., discriminating) of the two but is quite sensitive to deviations from normality in the parent populations.[2] Therefore, both tests were applied.

The Cochran test[3] uses a testing statistic "g" which is defined as:

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[1]See Acheson J. Duncan, Industrial Statistics and Quality Control (3rd ed.; Homewood, Illinois: Irwin, 1965), pp. 617-18, for a discussion of these assumptions and the consequences of violating them.

[2]Bernard Ostle, Statistics in Research (2nd ed.; Ames, Iowa: Iowa State University, 1963), p. 338.

[3]Albert H. Bowker and Gerald J. Lieberman, Engineering Statistics (Englewood Cliffs, N.J.: Prentice-Hall, 1959), p. 198.

$$g = \frac{\text{largest } S^2}{S_1^2 + S_2^2 + S_3^2 + \cdots S_k^2}$$

where

$$S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

These S^2 values are the unbiased estimates of the population variance for each cell. Tables are available for critical values of "g" at various levels of significance.[1] The parameters of the "g" distribution are the sample size "n" and the number of samples "k". If the computed "g" is larger than the critical "g", the null hypothesis of homogeneity of variance is rejected.

The Bartlett test[2] uses a testing statistic X^2 which is defined as:

$$X^2 = f \log_e S^2 - \sum f_i \log_e S_i^2$$

where

S^2 denotes the pooled estimate of variance from all groups,

$f_i = n_i - 1,$

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[1] Ibid., pp. 196-97.

[2] C. C. Li, Introduction to Experimental Statistics (New York: McGraw-Hill, 1964), pp. 438-39.

$$f = \sum f_i = N - k \text{ for } k \text{ groups,}$$

and

S_i^2 is the same as in the Cochran test.

The statistic X^2 is roughly distributed as χ^2 with k-1 degrees of freedom. A better correspondence to the χ^2 distribution can be obtained by dividing the X^2 value by the correction factor 1+c where

$$c = \frac{1}{3(k-1)} \left(\sum \frac{1}{f_i} - \frac{1}{f}\right)$$

If the corrected value of X^2 is greater than the critical χ^2 value, the null hypothesis of homogeneity of variance is rejected.

The results of both the Cochran and Bartlett tests are shown in Table 6.2. Since the test results for the original data indicated that the variances were significantly different, several transformations were attempted in an effort to find one which might bring the variances into line. For example, if variances are proportional to the mean, a new data set made up of the square roots of the original data will have equal variances.[1] In this case three transformations--log, square root, and reciprocal--were attempted and these results are also shown in Table 6.2

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[1]Ostel, Statistics in Research, p. 340.

TABLE 6.2

COCHRAN AND BARTLETT RESULTS
.01 LEVEL OF SIGNIFICANCE

	Cochran		Bartlett	
	Computed	Critical	Computed	Critical
All Twelve Systems				
Original Data	.503	.076	735.4	144.0
Log Transform	.288	.076	437.5	144.0
Square Root Transform	.171	.076	460.3	144.0
Reciprocal Transform	.664	.076	795.0	144.0
Six BASIC Systems				
Original Data	.800	.137	422.6	79.8
Log Transform	.449	.137	270.5	79.8
Square Root Transform	.345	.137	260.4	79.8
Six FORTRAN Systems				
Original Data	.343	.137	305.7	79.8
Log Transform	.241	.137	158.7	79.8
Square Root Transform	.255	.137	199.3	79.8

Obviously, even using the transforms, the differences in the cell variances are statistically significant. None of the computed statistics are remotely close to their critical values at the .01 significance level. These results could reflect, at least in part, that significant deviations from normality are present. However, this would also violate one of the ANOVA assumptions.

The general conclusion must be that the data of this study does not conform to the assumptions of an ANOVA test; therefore we must examine some alternative statistical tests.

6.4 Aspin-Welch Test

The Aspin-Welch (A-W) test[1] is a parametric test which offers some help under the conditions present in this particular study even though the assumptions underlying the test are not strictly met. The A-W test is designed to test for statistically significant differences between the means of two normally distributed populations which have unknown and possibly unequal variances. This is in contrast to the usual "t" test which assumes that the population variances are unknown but equal.

The Aspin-Welch test uses a testing statistic "t" which is approximately distributed as the "Student t"

[1]Duncan, Industrial Statistics, pp. 505-07.

distribution with "v" degrees of freedom.

$$t = \frac{\overline{X}_1 - \overline{X}_2}{S_1^2/n_1 + S_2^2/n_2}$$

where

$\overline{X}$ denotes the sample mean,

S^2 denotes the estimate of population variance,

and

n denotes the sample size.

In the usual "t" test, the number of degrees of freedom is equal to $n_1 + n_2 - 2$. For the A-W test, the number of degrees of freedom "v" is determined in the following manner:

$$v = \frac{1}{\dfrac{c^2}{n_1 - 1} + \dfrac{(1-c)^2}{n_2 - 1}}$$

where

$$c = \frac{S_1^2/n_1}{S_1^2/n_1 + S_2^2/n_2}$$

If the sample sizes and/or the variance estimates are more than slightly unequal, the A-W test produces a lower number of degrees of freedom than the usual "t" test. The lower the number of degrees of freedom, the higher the critical value of "t" becomes. Higher critical "t" values make it more difficult to reject the null hypothesis of equality of means.

The A-W test can be applied to our data if we forego the information content present in the segmenting of the data by task. In other words, we must view the three observations of each of nine tasks as merely being 27 items in an unsegmented sample. The effect of doing this is that we obtain a sample size of 27 which is close to 30, the sample size thought of as "large" for this test. When the sample size is 30 or over, the distribution of all possible sample means is approximately normal regardless of the distribution of the parent population.[1] Therefore, the A-W test should give a reasonably relevant test even though the assumptions are not strictly met. The major consideration is that the test does provide for the substantial inequalities in variance.

Since the A-W test only tests two samples at once, the test must be performed once for each combination of the systems being tested. For example, to run pair-wise tests on twelve systems requires 66 separate tests while six systems requires 15 tests. The results of applying the A-W test to our various data sets is shown in Tables 6.3, 6.4, and 6.5. In each of these tables, the systems are placed in order according to their average total cost. The least

[1]There is no practical way of testing whether these 27 items come from a normally distributed population.

cost system is at the left on the horizontal scale and at the top on the vertical scale. Therefore, if we divide any of these tables into quadrants, the upper left quadrant gives comparisons of least cost to least cost systems; the lower right quadrant, highest cost to highest cost; and the remaining two quadrants, least cost to highest cost and vice versa. We would expect then to find significant comparisons in the lower left and upper right quadrants.

Another way to view these tables is to examine each row (or column) searching for groups of non-significant results. Any such groups represent systems whose prices are not statistically different when compared to the "row" system. For example, Table 6.3 shows that compared to system B eight systems--G, C, K, I, J, D, F, and A--are statistically non-significant.

Table 6.3 gives the results of the A-W test applied pair-wise to all twelve systems. Thirteen pairs of systems proved significant at various standard significance levels with only the B-H and B-L comparisons giving unusual results. These two pairs proved significant because the variances involved happened to be unusually small.[1] The other eleven significant pairs are in the lowest vs. highest sectors as would be expected.

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[1]Systems B, H, and L were the three lowest variance systems at $12.31, $5.21, and $3.74 respectively compared to an average variance of $31.47 for all twelve systems.

TABLE 6.3

ASPIN-WELCH RESULTS FOR
ALL TWELVE SYSTEMS

	H	L	G	C	B	K	I	J	D	F	A	E
H	-				1				2	1	1	2
L		-			1				2	1	1	2
G			-									1
C				-								1
B	1	1			-							1
K						-						
I							-					
J								-				
D	2	2							-			
F	1	1								-		
A	1	1									-	
E	2	2	1	1	1							-

1 - significant at .05 level
2 - significant at .01 level
3 - significant at .001 level

When the twelve systems were separated into the six BASIC and six FORTRAN systems, some quite interesting results were obtained. No table is presented for the six BASIC systems because there were no pairs for which the A-W test produced significant results at even the .05 level. The FORTRAN systems, shown in Table 6.4, produced seven significant comparisons including the two unusual ones mentioned above. The remaining five were again clustered in the lowest-highest areas. Obviously any row comparisons show small non-significant FORTRAN groups compared to the BASIC systems which had no significant pairs at all.

When the vendors were compared as shown in Table 6.5, only two significant pairs were found. These were the two lowest cost systems compared to the highest cost system. Excluding the highest cost system, there were no significant comparisons among the remaining five systems.

We have only examined the statistical results of applying the A-W test and not the economic implications of these results. Since we have some more testing possibilities to examine, we will only survey in this chapter the results of these tests and base our statistical conclusions upon the composite results of the various tests. In Chapter VII, we will draw our economic conclusions based upon our overall statistical conclusions.

TABLE 6.4

ASPIN-WELCH RESULTS FOR SIX FORTRAN SYSTEMS

	H	L	B	J	F	E
H	-		1		1	2
L		-	1		1	2
B	1	1	-			1
J				-		
F	1	1			-	
E	2	2	1			-

1 - significant at .05 level
2 - significant at .01 level
3 - significant at .001 level

TABLE 6.5

ASPIN-WELCH RESULTS FOR SIX VENDORS

	X	Z	Y	V	U	W
X	-					1
Z		-				1
Y			-			
V				-		
U					-	
W	1	1				-

1 - significant at .05 level
2 - significant at .01 level
3 - significant at .001 level

6.5 Friedman Multi-Sample Test

Since we cannot state without qualification that the data of this study conforms to the assumptions of parametric type tests, we should also examine the data using some nonparametric or distribution free statistical tests.[1] Before examining the first of these tests, we must review carefully the differences between parametric and nonparametric tests with respect to the null hypothesis.

The differences in the null hypotheses are rather paradoxical. A typical parametric test of differences between population means might require that we assume the parent populations to be normally distributed with equal variances. The null hypothesis would then be that the means of the parent populations are equal. If the testing statistic falls outside the critical value, the null hypothesis is rejected; the conclusion is that the parent population means are different. This conclusion however is based upon the original assumptions. All other sources of difference have been assumed away. If the data does not precisely conform to the assumptions, then other sources of difference are really present; they are simply being ignored explicitly and are being lumped implicitly with the differences between

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[1]Technically speaking, these two terms are not synonymous, but the distinction is maintained only by theoretical statisticians. See James V. Bradley, Distribution-Free Statistical Tests (Englewood Cliffs, N.J.: Prentice-Hall, 1968), p. 15.

means.[1]

For a typical nonparametric test, the null hypothesis is only that the distributions are the same in all characteristics. Rejection of the null hypothesis yields rather ambiguous conclusions. There is no way of knowing whether the form of the distributions, the means, or the variances were unequal.[2] However, acceptance of the null distribution conveys more information in the nonparametric case. Here, the distributions are accepted as equal in all aspects without equality having been assumed (perhaps incorrectly) for most of the aspects.

The nonparametric equivalent of the ANOVA test is the Friedman multi-sample test.[3] Although we will require a modified version which allows replications, the test can best be explained by examining the basic case. Suppose that we have three treatments (columns) which we apply to each of four separate blocks (rows) giving us a total of twelve observations. Suppose also that the treatments are identical with respect to their effect upon each block. If we were to rank the observations in each row, we would expect the rank sums for the columns to be approximately

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[1] Ibid., Chap. 1-2.

[2] Ya-lun Chou, Statistical Analysis (New York: Holt, Rinehart, and Winston, 1969), pp. 476-77.

[3] Bradley, Distribution-Free Statistical Tests, pp. 123-29.

equal since the rank values would occur randomly in each column if the treatments were really identical in effect. A statistical test would consist of comparing the actual column rank sums to the average for all columns.

TABLE 6.6

EXAMPLE OF FRIEDMAN TEST

	Original Data				Ranked Data		
	A	B	C		A	B	C
1	7.2	5.3	6.5	1	3	1	2
2	1.1	3.4	8.6	2	1	2	3
3	5.7	4.2	9.0	3	2	1	3
4	.3	8.5	4.8	4	1	3	2
				Total	7	7	10

Table 6.6 shows a small example of the tabular procedures in the basic Friedman test. If the null hypothesis were to be exactly satisfied, the three rank sum totals would all equal eight. The basic Friedman test tells us whether the differences in the rank sums are too great to be attributable to chance. We will not go through the computational procedures for the simple case since we must use the more complex form.

In our case, the systems are the treatments, and the tasks are the blocks. However, we do not have a single observation per cell or block-treatment intersection.

Instead, we have three observations for each cell. The method of handling replications (assuming equal observations per cell) is to consider all the observations within the block as one group and rank them. Now, each column has three ranks per block in our case. Again, if the null hypothesis is true, the contributions to the rank sums should be randomly distributed.

The method described above is the way in which rankings are assigned in theory. In practice, another method is used which lends itself better to computerization for larger problems.[1] This alternate method involves computing a modified rank value for each observation in a block. Each observation is compared to all the other observations in its block except those in its own column. For each comparison, the rank value is increased by one if the base value is larger and decreased by one if the base value is smaller. The effect of this is to construct differential rankings about a mean of zero. The testing statistics are modified somewhat from what they would be if the actual ranks were used; however the eventual outcomes are identical.

This test uses a testing statistic "T" as shown below:

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[1]Gottfried E. Noether, Elements of Nonparametric Statistics (New York: Wiley, 1967), pp. 52-53.

$$T = \frac{3}{Cn^2(Cn+1)M} \sum_{i=1}^{C} W_i^2$$

where

C denotes the number of columns,

M denotes the number of blocks,

n denotes the number of replications,

and

W_i denotes the column sum of the modified rank values as described above.

"T" is distributed approximately as the χ^2 distribution with C-1 degrees of freedom.

The Friedman test was applied to all four data sets with the results shown in Table 6.7. In every case, the null hypothesis of equality of distribution from system to system was easily rejected at the .001 level of significance. We must remember however that this does not prove that the average costs for the systems are different, only that the distributions are different in one or more aspects.

Since the Friedman test results are so striking, we must run some type of pair-wise nonparametric tests to attempt to determine where the differences lie. Further testing will not help in determining the source of variation in terms of means or variances; however, it will allow us to find where differences do not exist.

TABLE 6.7

FRIEDMAN TEST RESULTS .001
LEVEL OF SIGNIFICANCE

	Computed	Critical
All Twelve Systems	200.5	31.3
Six BASIC Systems	77.7	20.5
Six FORTRAN Systems	113.5	20.5
Six Vendors	108.2	20.5

6.6 Wilcoxon Rank Sum Test

The nonparametric pair-wise test which we will use is called the Wilcoxon rank sum test with randomized blocks.[1] Again, the null hypothesis is that the distribution of average costs is the same for the two systems being tested.

The testing statistic "T_ε" is computed as shown below:

$$T_\varepsilon = \frac{1}{n^2} \sum_{i=1}^{M} W_i$$

where

M denotes the number of blocks,

n denotes the number of replications,

[1]Noether, Nonparametric Statistics, pp. 41-43.

and

W_i denotes the modified rank value for a block as computed for the Friedman test.

Note that the rank sum statistic for only one column is used; the sum for the other column would simply be equal in magnitude but opposite in sign to the sum of the first column. "T_ε" is approximately normally distributed with a mean of zero and a variance as shown below:

$$\text{var } T_\varepsilon = \frac{M(2n+1)}{3n^2}$$

where

M denotes the number of blocks,

and

n denotes the number of replications.

To test using "T_ε", we must divide "T_ε" by its standard deviation (i.e., the square root of its variance) and compare the value obtained to the critical normal value.

The Wilcoxon test produced the results shown in Table 6.8 when it was applied to the data of all twelve systems. It is readily apparent from the density of the entries in this table that the null hypothesis is rejected for nearly every pair. Also, many of the table entries are "3"s meaning that the rejection is at the .001 significance level.

TABLE 6.8

WILCOXON RANK SUM TEST
ALL TWELVE SYSTEMS

	H	L	G	C	B	K	I	J	D	F	A	E
H	-	2	2	3	3	2	2	2	3	3	3	3
L	2	-	3		3		1	1	3	3	3	3
G	2	3	-	2	3	3	3	3	3	3	3	3
C	3		2	-		2		1	3	3		3
B	3	3	3		-	3		2	1	1		3
K	2		3	2	3	-	2		3	3	3	3
I	2	1	3			2	-		3	3	2	3
J	2	1	3	1	2			-	3	3	2	3
D	3	3	3	3	1	3	3	3	-	3		3
F	3	3	3	3	1	3	3	3	3	-		3
A	3	3	3			3	2	2			-	2
E	3	3	3	3	3	3	3	3	3	3	2	-

1 - significant at .05 level
2 - significant at .01 level
3 - significant at .001 level

The Wilcoxon results for the BASIC systems are shown in Table 6.9. Only three of the fifteen comparisons proved to be non-significant, and there seems to be no readily apparent pattern in their positions.

TABLE 6.9

WILCOXON RANK SUM TEST
SIX BASIC SYSTEMS

	G	C	K	I	D	A
G	-	2	3	3	3	3
C	2	-	2		3	
K	3	2	-	2	3	3
I	3		2	-	3	2
D	3	3	3	3	-	
A	3		3	2		-

1 - significant at .05 level
2 - significant at .01 level
3 - significant at .001 level

The Wilcoxon results for the FORTRAN and Vendor systems are not shown in tabular form because of the uniformity of the results in each case. For the FORTRAN systems, all comparisons were statistically significant, ten of fifteen at the .001 level of significance. For the vendor systems, only the U-V pair was statistically non-significant. Again, ten of fifteen comparisons proved statistically significant at the .001 level.

The results of the preceding applications of the Wilcoxon rank sum test can only be summarized as indicating highly significant differences in distribution throughout all four data sets. We must suspect that a significant portion of these differences are arising from the differences in variance which were discovered earlier.

6.7 Additional Analyses

Since the preceding tests uncovered numerous statistically significant differences, one additional set of tests was conducted. The Aspin-Welch, Friedman, and Wilcoxon tests were again applied to two additional data sets, both based upon all twelve systems. These two data sets are: (1) the running charges, and (2) the running charges plus the storage charges. These two were chosen because they represent additional possible (although inferior) system selection criteria for buyers. In other words, buyers might only be exerting pressure in the market for vendors to equalize their running charges or their running plus storage charges.[1] For example, a buyer who ignored labor costs would be basing decisions only upon running and storage charges.

The results for these revised data sets were not very different from those obtained for the original data set. The Aspin-Welch and Friedman test results both

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[1]In the author's opinion, there is an inexplicable tendency among many buyers to dismiss costs other than the running charges as less important. This may occur because users are effectively reminded of running charges by the performance statistics printed out each time they use the system.

indicated that the differences were slightly less for the running plus storage charges and still less for the running charges only. However, the differences were very slight. The Wilcoxon results were quite inconclusive relative to the original results.

The only relevant conclusion possible from these extra tests is that running charges do seem to be slightly closer to equal across all systems than the average total costs are. Nevertheless, approximately the same number of significant differences between systems are found regardless of whether storage charges and labor costs are included.

6.8 General Statistical Conclusions

It is unfortunate that the observed data does not fit some standard statistical test which will in turn provide a simple yes or no answer to the null hypothesis of the economic model. However, the almost inevitable fate of the empirical researcher in the social sciences is to encounter situations similar to what we have in this study. Also, the more complex the relationships under investigation are, the more likely these situations are. We can only attempt to extract as much information as possible from the variety of statistical tests used.

Each statistical test used (with one exception) has revealed significant differences of one sort or another for all data sets. The Aspin-Welch test did fail to find a significant pair among the BASIC systems. However, the general pattern is unquestionably one of significant differences. The results of the parametric Aspin-Welch test are reinforced by the results of the non-parametric Friedman and Wilcoxon tests despite the differences in hypothesis structure discussed earlier.

The null hypothesis of average cost equality among all systems, among the FORTRAN systems, and among the vendors must be rejected. The BASIC systems do present somewhat of a problem since the non-parametric tests indicate differences while the parametric test does not; therefore, we are left with the rather unsatisfactory conclusion that we should not reject the null hypothesis for the BASIC systems, but that we strongly suspect inequalities may exist.

For all the data sets, we can state that the cost distributions vary significantly for almost all system comparisons. While this is not a direct answer to the original null hypothesis, it does tell us something about the complexity of the selection decision facing a buyer. The systems definitely don't have similar distributions and variances leaving only differences in arithmetic means for the buyer to consider. Instead he must examine all aspects of the distribution of costs of using a system.

6.9 Summary

We have examined the problems involved in statistically analyzing the data of this study. We were forced by the data to discard the most attractive potential tool, the analysis of variance, and to adopt some less conclusive tests. Nevertheless, the preponderance of evidence leads us to reject the null hypothesis of the study and to conclude that effective system prices (and consequently buyer costs) are different.[1]

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[1]A commercial time-sharing system was used for the data preparation, the statistical analyses, and the generation of Appendices IV through VII. The entry, debugging, and execution of the programs necessary for these tasks took approximately 24 hours of terminal connect time and over 7 minutes of CPU time. The total cost was approximately $325. These figures should give the reader some perspective on the costs of performing "real world" tasks from beginning to end.

CHAPTER VII

THE CONCLUSIONS

7.1 Introduction

This is the final and most important chapter for, here, we will examine the economic conclusions of this study. However, before proceeding, we will review briefly the steps we have taken up to this point.

The basic problem has been the determination and comparison of the effective prices being charged for a fairly new and technical product which has a rather complex nominal pricing structure. The entire study has been constructed along the traditional lines of an empirical experiment. Therefore, the first step was a review of the general problem followed by a review of the economic theory relevant to this problem, namely, the theory of differentiated oligopoly. The next step was to examine in detail any characteristics of the time-sharing market which would help in determining the level of competition to be expected in market pricing. Based upon this examination, the null hypothesis was formed that effective prices in the market are equal.

The experiment to test this hypothesis forms the core of this study. An entire methodology for measuring effective prices had to be developed. This methodology consisted of defining and actually developing a unit of product for computer time-sharing, developing standardized procedures for obtaining this unit of product, and selecting the vendors whose product would be tested. After the experiment was conducted, the data obtained was analyzed using a combination of parametric and nonparametric statistical tests. In general, it was found that the null hypothesis of price equality was rejected.

We will now proceed with the economic conclusions of the study starting with the implications of the findings we have already obtained. Then, since we have effectively assumed away all qualitative differences between systems (except those performance differences measured by our methodology), we must relax this assumption and determine whether any non-measured differences in quality either alter or explain our empirical conclusions. Finally, we will examine some public policy implications of the study.

7.2 The Rejection of the Null Hypothesis

The null hypothesis of the study (as originally stated in Section 3.11) is as follows:

> Time-sharing services are sold to an economically motivated and analytically sophisticated group of customers in a market which is apparently fairly

competitive; therefore, the apparent disparities present in the nominal prices are equalized by differences in performance such that effective prices are equalized.

The composite results of the several statistical tests used indicated clearly that the null hypothesis must be rejected (see Section 6.8).

If we could have accepted the null hypothesis, we could have concluded that competitive pressures (e.g., a low degree of product differentiation coupled with freedom of entry) in the market place caused effective price equality. As discussed in Section 3.11, price equality could conceivably indicate other types of market conduct such as collusion or, more likely, price leadership. However, because of the newness of the industry, the multitude of regional competitors, and the continual addition of new competitors, the establishment of the quasi-agreements necessary for effective price leadership would be quite difficult at this point in the industry life cycle. Therefore, we would have accepted the null hypothesis as it was stated.

Our analysis could end at this point *if* we could have accepted the null hypothesis. Our conclusions would then have been those described above. However, since we have rejected the null hypothesis, we must examine the implications of accepting the alternate hypothesis. The alternative hypothesis (again from Section 3.11) is:

> Conditions in the market place are such that distinct product differentiation can and does exist as manifested by significantly different effective prices being charged to accomplish the same tasks with a similar level of performance.

Since we have found effective prices to be different for products we have assumed to be identical, this is direct evidence of product differentiation. In fact, it is evidence of what we termed "meaningless" differentiation in Section 2.5. The implication of this differentiation is, of course, that certain sellers have sufficient monopoly power that they are able to charge higher prices for the same product. As we indicated in Section 3.8, the most probable source of this monopoly power is buyer ignorance with imperfect buyer mobility possibly being a contributing factor.

Although time-sharing as a product is quite homogeneous, perfect product homogeneity is an assumption which we made to simplify the analysis. Therefore, we will now relax this assumption.

7.3 Prices Relative to Vendor Quality Differences

All of the systems are not identical; therefore, we must consider the possibility that "meaningful" product differences do exist and do explain the differences in buyer cost which we have discovered. So that we may make a clear distinction, let us call those differences in quality which we have not yet considered the qualitative differences in systems. We are really looking at those quality differences other than speed. The systems can vary in numerous ways; however, the two most important factors which we have not considered are the system library and the general operating system[1] capabilities.

System libraries can vary both in size and quality, including the quality of the documentation (e.g., manuals) available to the would-be user. Size must be viewed with the concept of usefulness kept in mind. Anyone can build a huge library of programs which is of little interest or usefulness; however, building a library of good general-purpose programs is slow and expensive. When we speak of libraries, we are including only those programs available to the system user at no extra charge. A few vendors have unique, highly specialized library programs available[2] at

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[1]The operating system is the master program which oversees or supervises every facet of the time-sharing systems operations.

[2]Typical examples would be numerical control machine tool programs or specialized engineering programs.

extra charge; these vendors are effectively monopolists[1] with respect to buyers who need these unique programs.

Similarly, a good time-sharing operating system is one which has a large number of facilities which are useful to the users. For example, the ability to store programs in compiled form and the ability to edit program statements without complete re-entry would be two very useful system facilities. Most time-sharing systems have the same basic facilities, but there is considerable variety beyond these basics.

There are numerous other qualitative considerations in evaluating a time-sharing service, such as the system's reliability and the quality of support personnel. Factors such as these are quite difficult to measure or even subjectively appraise and are also very subject to change. Therefore, we will restrict our evaluation to the two major considerations of library and operating system quality.

Table 7.1 shows the author's ranking[2] of the six vendors with respect to their libraries and their overall operating system capabilities. These ranks were not

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[1]If one or two other vendors have close substitutes, we may have a small oligopolistic market for the particular program.

[2]Although the opinions of actual users might be slightly more accurate, it was impossible to find users who had had experience on more than three of the included systems. This effectively eliminated a questionnaire type approach.

particularly difficult to assign as the decisions in most cases were clear-cut.[1] On the other hand, determining cardinal values to represent quality would be virtually impossible. Since both characteristics are virtually identical for both of the vendor's languages (i.e., systems), it is only feasible to make these rankings by vendor. Also, in establishing these rankings, any differences which were measured in the quantitative portion were ignored to avoid "double counting." Additional factors not considered were the vendor's name, image, or market share.

Table 7.2 shows the composite qualitative ranking,[2] along with an average total cost ranking based upon the data of Table 6.1.

If qualitative differences are to explain the differences in vendor cost, then the vendors with the better overall quality rankings should be the high cost vendors. Similarly those with the lower quality systems should be the low cost vendors. In terms of Table 7.2, this means

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[1]The rankings would probably seem quite difficult to assign from the viewpoint of an inexperienced user. However, the author spent over 30 connect hours on each time-sharing system and over a hundred hours on several of them. In addition, at least another 30 hours were spent examining the manuals and other publications of each system. Another valuable source of information was the comments of experienced users of each system.

[2]The library and operating system components were given equal weights in making up the composite rankings. Neither is predictably more important to the typical user.

TABLE 7.1

QUALITATIVE VENDOR RANKINGS

Vendor	General Library	Operating System
U	5.5	6
V	1	3
W	4	4
X	4	2
Y	2	1
Z	5.5	5

Rankings are from best (1) to worst (6)

TABLE 7.2

VENDOR QUALITY AND COST RANKINGS

Vendor	Quality	Cost
U	6	2
V	2	3
W	4	1
X	3	6
Y	1	4
Z	5	5

Quality rankings are from best (1) to worst (6). Cost rankings are from highest (1) to lowest (6).

that there should be a high degree of correlation between the quality and cost ranks. In fact, the correlation is rather poor.[1] Vendor U has the lowest quality and second highest cost while vendor W has the fourth highest quality and the highest cost. There is nothing in the above findings to support the view that the cost differences are explained away by qualitative differences between vendors.

7.4 Prices Relative to Vendor Image and Market Share

The relationship between prices and vendor image and/or market share is rather difficult to evaluate. Two of the six vendors are large nationwide vendors and are divisions or subsidiaries of computer manufacturers (see Section 5.4). If these two vendors were the highest in price and low or medium in quality, we might conclude that their images and market positions allowed them to charge higher prices for the same or lower quality products (i.e., image and/or market share contribute to considerable product differentiation). On the other hand, if these two firms are lowest in price and highest in quality, we might conclude that economies of scale are a very significant force in the market. The large firm has a much broader base over which to spread the costs of developing a higher

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[1]The coefficient of rank correlation is -.26 which is indicative of virtually no relationship (i.e., only 7 per cent explained variance).

quality library and operating system.

Neither of these two extreme patterns can be determined from our results. Instead, we find that the two largest firms, vendors Y and V, are third and fourth in effective pricing level and first and second in overall quality. Evidently, the larger firms are providing the best quality service while charging only medium prices. From the buyer's point of view, he can use the highest quality systems for only medium prices. However, if he does not need the extra quality, he can buy a lower quality product at lower prices from vendors X or Z.

7.5 General Conclusions About Industry Prices

Based upon the results of our various analyses, we can state the following conclusions about prices in the computer time-sharing industry:

1. There appear to be significant differences in the effective prices charged for a typical unit of product. Expressed in another way, the variations in nominal prices do not tend to reflect accurately the differences in hardware and software capabilities which exist from firm to firm.
2. There appear to be significant differences in the effective prices regardless of whether

vendors, FORTRAN systems, BASIC systems, or BASIC and FORTRAN systems together are compared. There does appear to be far more variability in the effective prices among the FORTRAN systems than among the BASIC systems.[1]

3. Differences in vendor quality (other than speed) do not tend to explain the differences in effective prices. In fact, there is virtually no correlation between effective price level and system quality.

4. The effective prices of the large nationwide vendors are in the middle of the price range while they have superior quality libraries and system facilities.

5. Time-sharing services can be obtained at lower effective prices than those charged by the largest vendors if the buyer is willing to forego some quality.

Obviously, the applicability of this study to the entire time-sharing industry is a function of both the validity of the methodology and the representativeness of

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[1]One reasonable explanation for the smaller amount of variability among BASIC systems is the fact that BASIC is a more uniform language from vendor to vendor than FORTRAN. BASIC was developed specifically for time-sharing use by Dartmouth College, and time-sharing firms have adopted it virtually intact, making only minor modifications.

sample of firms. The author can only state that, although many subjective decisions were made (as was obvious in Chapter V), every attempt was made to maintain an unbiased viewpoint. Someone else making the same study might have made different decisions at various points; however, it is hard to believe that the results would have varied significantly, using any reasonable set of assumptions. The author, therefore, is confident that the conclusions of this study are applicable to the industry as a whole.

7.6 Some Conclusions About Industry Structure and Conduct

As stated earlier, it is impossible to draw definitive conclusions about market conduct based solely upon the results of a single static analysis of industry prices. However, we may draw some reasonable inferences which could vary possibly serve as hypotheses for subsequent studies.

As we saw in Chapter III, the time-sharing industry is definitely oligopolistic. It is not only oligopolistic because two vendors have a large share of the national market but also because in each local area (where direct competition occurs) there is a limited number of competitors (e.g., less than ten except in the few largest cities). These local and regional competitors comprise the remainder of the approximately 200 total firms in the industry. Another important fact is that firms in this industry charge uniform prices in all markets which they serve (i.e., freight absorption). These factors combine to produce an interesting and somewhat unusual market structure. As stated above, any local market is oligopolistic since all sellers are well aware of the presence and actions of their competitors. However, the large nationwide firms are facing in total a host of competitors. Therefore, to the large firms, many regional or local firms may very well appear as atomistic when viewed on a national basis but as oligopolistic when viewed on the local basis where market confrontation actually occurs. However, since, for the large firms, most policy seems to be set at the national level, the ability of these large firms to respond is somewhat limited at the local level. This is a special case of the Fellner and Bain models of partial oligopoly as discussed in Sections 2.3 and 2.4. About the only conclusion we can draw here is that the smaller firms are able to exist and prosper (at present) as significant competitors in local or regional markets.

At present, there appears to be a low-to-moderate degree of product differentiation in the time-sharing industry. As stated earlier, the sophisticated customer is presently a heavy user of the basic system capabilities and is primarily a purchaser of raw time-shared computer power which is a rather uniform product with very little physical differentiation. Nevertheless, buyer ignorance (or possibly, other unconsidered factors) is allowing some firms to charge higher prices for somewhat inferior products. This is empirical evidence of product differentiation although the source cannot be definitely pinpointed. If the level of product differentiation does rise, especially in the area of significant improvements in libraries and systems, then the ability of smaller firms to compete (and enter) may be seriously hampered. Here, the question of economies of scale becomes very significant. Unfortunately, we are still lacking solid information in this area.

Based upon the foregoing discussion and other details discussed in Chapter III, the author suspects that the firms in the market fall into the following categories:

1. The major firms in the industry who set the reference level for effective prices and quality. These firms are extremely "visible" to one another and probably have already established Fellner-type quasi-agreements with one another despite the immaturity of the industry. Their quasi-agreements would appear at this time to cover prices only with constant product changes being outside the agreement.
2. The aggressive local and regional competitors who, realizing that they cannot afford to main-

tain the quality level of the majors, attempt to undercut the effective prices of the major firms for raw computer power. These smaller firms very possibly have quasi-agreements among themselves in their particular markets and, in addition, probably limit their behavior to actions which they feel will not provoke retaliation by the industry giants. However, as discussed earlier, the ability of the giants to retaliate against specific small competitors seems limited. This class of firms could vary from those who offer a little less quality at a slightly lower price to those who offer a lot less quality at a greatly reduced price. The latter would be the vendors of completely raw computer power.

3. The marginal firms, low in overall quality and high in price, who survive by either fulfilling specialized needs, continually selling newcomers to the market, or employing unusually high levels of promotional or direct sales effort. These firms are marginal in the sense that they are the most susceptible to aggressive competition. A typical example could be the firm which was an early entrant to the market but which has

> failed to keep pace in product development. The rapid industry growth is undoubtedly allowing such firms to remain viable. Whether such firms could exist in an equilibrium market is another matter.

The first two types of firms taken together could produce the two "clusters" of firms which are common in differentiated oligopolies according to Bain (see Section 2.5). In fact, the pattern of industry development may very well be along these lines: that is, high quality-high price firms and low quality-low price firms. This may be the only answer for smaller firms if the cost of product changes becomes too high and if these product changes become product differentiation in the eyes of buyers. As stated above, the third type of firm will probably disappear as the industry matures.

This seems to be the extent to which we can comment on market conduct in the time-sharing industry based upon these study results. More knowledge of this conduct will have to await future studies; possibly one saving grace is that the industry is an interesting one to study based upon the author's experience.

7.7 The Implications for Public Policy

It is possible that the commercial computer time-sharing industry could become a regulated industry similar to the public utility industries. In fact, the term "computer utility"[1] has been used considerably in the industry. The probability of regulation occurring is dependent upon the direction and extent of industry growth. There are some factors which make regulation more probable and others which make it less probable.

When we speak of regulation, we are referring to the same type of regulation as is imposed upon public utilities. Such regulation could be imposed upon this industry to protect the public interest under any one or more of several conditions.

1. Technology might make practical a huge machine which could economically serve the computer needs of a large area (e.g., an entire state).[2] The public might best be served by allowing one such "computer utility" to operate in each area.
2. As the construction of data banks--especially those containing information about individuals--occurs, the protection of the rights of privacy,

[1]For example, see D. F. Parkhill, The Challenge of the Computer Utility (Reading, Mass.: Addison-Wesley, 1966).

[2]The "square law" is valid for a given state of technology (i.e., for a given point in time). There are computers currently under development which have potentials that stagger the imagination.

etc., might lead to close supervision and/or regulation. Also, duplication of stored information is just as inefficient as the duplication of physical facilities.

3. The time-sharing industry is inextricably tied in a physical sense to the general communications industry which is heavily regulated. If either moved more directly into the province of the other, the time-sharing industry could become regulated.

There are, however, opposing forces to all of the above three.

The most troublesome aspect in predicting the future pattern of development in the time-sharing industry appears to be the uncertainty of the future of communications costs and capabilities. Already, time-sharing has contributed to telephone service crises in major cities, particularly New York.[1] Many see communications costs as the most limiting factor in time-sharing growth.[2] In other words, communications costs might nullify many of the economies of scale possessed by large centralized computer systems. In fact, companies may find it cheaper to purchase

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[1]See Dan Cordtz, "The Coming Shake-Up in Telecommunications," FORTUNE, April, 1970, p. 69, for a discussion of the many troubles currently plaguing the Bell system.

[2]Guise, "The '69 Gold Rush," p. 41.

their own smaller, less-efficient time-shared computers. The lower efficiency may be more than offset by the reduced communications costs of private wiring (i.e., not using the public communications systems at all). Certainly, improvements will occur in public communications; however, it is estimated that altering the basic structure of the telephone network would require over ten years plus billions of dollars.[1] The general computer utility may be more of a technical possibility than an economic reality for the reasonable future.

The economic feasibility of central data banks is also open to question. There are only a limited number of types of information which are needed on a virtually instantaneous basis. We are talking now of data banks which would be useful to large segments of the general business community. Obviously, specialized applications such as airline reservation systems are now operating. For many types of data, receiving a tape once each month is completely adequate in terms of updating information. The potential of an "information utility" may also be vastly over-rated.

Finally, the question of whether the communications utilities invade the time-sharing industry or vice-versa

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[1]Peter L. Briggs, "Future Time-Sharing Technology," COMPUTERWORLD, March 25, 1970, p. S10.

will ultimately be a public policy decision set by law or regulatory agencies. Thus far, the divisions between industries are remaining fairly stable; however, competition is arising for AT&T from microwave carriers who intend to specialize in inter-city data transmission.[1] Whether there will be any strong moves across industry lines remains to be seen.

Whether or not this industry will become regulated is obviously open to considerable conjecture at this point. Even if it does not, it is still subject to normal antitrust regulation.[2] If the industry does come under either regulation or close antitrust inspection, the results of this and subsequent studies may provide some quite relevant data on the effective prices in the market place. For example, from an antitrust standpoint, the conclusions of Section 7.4 do not appear to reflect badly upon the relative performance of the nationwide vendors in terms of value for price. However, if subsequent analyses showed extraordinary similar movements in effective prices, this _might_ raise suspicions of collusion. In a regulatory environment, the

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[1]"The Round AT&T Lost," _Business Week_, September 6, 1969, p. 68.

[2]The Service Bureau Corporation was originally created by IBM in response to Justice Department pressure. SBC took over the IBM time-sharing division in 1968 to forestall another application of such pressure to its parent.

conduct of the marginal firms referred to in Section 7.6 would be very open to question (i.e., low quality and high prices).

This would be a difficult industry to regulate and even to simply maintain a watch upon because of the measurement difficulties discussed throughout this study. Furthermore, because of the many problems involved, a better (i.e., more informative) nominal pricing structure will probably never be developed.

7.8 Summary

Based upon the statistical results of the study, the null hypothesis of effective price equality must be rejected. If the assumption of completely homogeneous products is relaxed, the differences in quality do not tend to explain the differences in effective prices. Furthermore, considering vendor image or market share does not explain the differences either. The conclusion is that product differentiation does exist with the probable cause being buyer ignorance.

The industry is definitely a differentiated oligopoly but with several unusual characteristics caused by geographical considerations and industry pricing policies. The present structure is typified by three major classes of firms but is likely headed toward two classes: high quality-high price vendors and lower quality-lower price firms. The large nationwide vendors will probably be the former type and the local or regional vendors the latter type.

It is possible that the industry could become a regulated one of the public utility type. If this does occur, the information supplied by this and subsequent similar studies would be of significant benefit and interest to those responsible for this regulation.

APPENDIXES

Number

```
1000 REM  *** T1B ***
1010 LET M = 0
1020 LET I1 = 50
1030 LET J1 = 20
1040 LET K1 = 100
1050 FØR I = 1 TØ I1
1060 FØR J = 1 TØ J1
1070 FØR K = 1 TØ K1
1080 LET M = M + 1
1090 NEXT K
1100 NEXT J
1110 NEXT I
1120 PRINT
1130 PRINT "TEST 1     BASIC"
1140 PRINT
1150 PRINT "INPUT - 0    CPU - H    ØUTPUT - L"
1160 PRINT
1170 PRINT "I = "; I1; "  J = "; J1; "  K = "; K1
1180 PRINT
1190 PRINT "M = "; M, "     END TEST 1"
1200 END
```

```
1000 REM  *** T2B ***
1010 LET M = 0
1020 LET I1 = 50
1030 LET J1 = 20
1040 LET K1 = 100
1050 PRINT
1060 PRINT "TEST 2    BASIC"
1070 PRINT
1080 PRINT "INPUT - 0    CPU - H    ØUTPUT - H"
1090 FØR I = 1 TØ I1
1100 PRINT
1110 PRINT "THIS IS LINE NUMBER "; I
1120 FØR J = 1 TØ J1
1130 FØR K = 1 TØ K1
1140 LET M = M + 1
1150 NEXT K
1160 NEXT J
1170 NEXT I
1180 PRINT
1190 PRINT "I = "; I1; "  J = "; J1; "  K = "; K1
1200 PRINT
1210 PRINT "M = "; M, "     END TEST 2"
1220 END
```

```
1000 REM   *** T3B ***
1010 LET M = 0
1020 LET I1 = 50
1030 PRINT
1040 PRINT "TEST 3     BASIC"
1050 PRINT
1060 PRINT "INPUT - O    CPU - L    ØUTPUT - H"
1070 PRINT
1080 FØR I = 1 TØ I1
1090 LET M = M + 1
1100 PRINT
1110 PRINT "THIS IS LINE NUMBER "; I
1120 NEXT I
1130 PRINT
1140 PRINT "M = "; M, "     END TEST 3"
1150 END
```

```
1000 REM   *** T4B ***
1010 LET M = 0
1020 LET I1 = 50
1030 PRINT
1040 PRINT "TEST 4     BASIC"
1050 PRINT
1060 PRINT "INPUT - H    CPU - L    ØUTPUT - H"
1070 PRINT
1080 FØR I = 1 TØ I1
1090 INPUT N
1100 LET M = M + N
1110 PRINT
1120 PRINT "THIS IS LINE NUMBER "; I
1130 NEXT I
1140 PRINT
1150 PRINT "M = "; M, "     END TEST 4"
1160 END
```

```
1000 REM   *** T5B ***
1010 LET M = 0
1020 LET I1 = 50
1030 PRINT
1040 PRINT "TEST 5     BASIC"
1050 PRINT
1060 PRINT "INPUT - H    CPU - L    ØUTPUT - L"
1070 PRINT
1080 FØR I = 1 TØ I1
1090 INPUT N
1100 LET M = M + N
1110 NEXT I
1120 PRINT
1130 PRINT "M = "; M, "      END TEST 5"
1140 END
```

```
1000 REM   *** T6B ***
1010 LET I1 = 10
1020 LET J1 = 1000
1030 PRINT
1040 PRINT "TEST 6    BASIC"
1050 PRINT
1060 PRINT "BUILT-IN FUNCTIØNS"
1070 PRINT
1080 FØR I = 1 TØ I1
1090 FØR J = 1 TØ J1
1100 LET X = ABS(SIN(INT(SQR(100 * RND(0)))))
1110 NEXT J
1120 PRINT
1130 PRINT "I = "; I; "  X = "; X
1140 NEXT I
1150 PRINT
1160 PRINT "     I = "; I1; "       J = "; J1; " END TEST 6"
1170 END
```

```
1000 REM   QUEUING SIMULATIØN PRØGRAM
1010 REM
1020 REM       SINGLE QUEUE - SINGLE CHANNEL
1030 REM
1040 DIM A(10), B(10), C(10), D(10)
1050 DIM M(10), N(10), Ø(10), P(10), Q(10)
1060 DIM R(10), V(10), W(10), X(10), Y(10)
1070 DIM E(400), F(400), G(400), H(400)
1080 DIM J(400), S(400), T(400), U(400)
1090 PRINT
1100 PRINT
1110 PRINT
1120 PRINT
1130 PRINT "            Q U E I N G   S I M U L A T I Ø N"
1140 PRINT "          <><><><><><><><><><><><><><><><><><><>"
1150 PRINT
1160 PRINT
1170 PRINT
1180 PRINT "     I N P U T   D A T A "
1190 PRINT "     ******************"
1200 PRINT
1210 PRINT
1220 PRINT "HØW MANY INTERARRIVAL TIME PØSSIBILITIES (1 TØ 10) ";
1230 INPUT A2
1240 IF A2 < 1 THEN 1270
1250 IF A2 > 10 THEN 1270
1260 GØTØ 1290
1270 GØSUB 3660
1280 GØTØ 1230
1290 PRINT
1300 PRINT
1310 PRINT "HØW MANY SERVICE TIME PØSSIBILITIES (1 TØ 10) ";
1320 INPUT A3
1330 IF A3 < 1 THEN 1360
1340 IF A3 > 10 THEN 1360
1350 GØTØ 1380
1360 GØSUB 3660
1370 GØTØ 1320
1380 PRINT
1390 PRINT
1400 PRINT "HØW MANY SETS ØF 100 SIMULATIØNS DESIRED (1 TØ 10) ";
1410 INPUT A4
1420 IF A4 < 1 THEN 1450
1430 IF A4 > 10 THEN 1450
1440 GØTØ 1470
1450 GØSUB 3660
1460 GØTØ 1410
1470 PRINT
1480 PRINT
1490 PRINT "DESIRED TIME PERIØD IN MINUTES ";
1500 INPUT A5
1510 IF A5 >= 1 THEN 1540
1520 GØSUB 3660
```

```
1530 GØTØ 1500
1540 PRINT
1550 PRINT
1560 PRINT "IN ASCENDING ØRDER, TYPE IN EACH INTERARRIVAL TIME"
1570 PRINT "PØSSIBILITY AND THE UPPER BØUND ØF THE ASSØCIATED"
1580 PRINT "RANDØM NUMBER SET ØN A LINE."
1590 FØR K = 1 TØ A2
1600 INPUT A(K), B(K)
1610 NEXT K
1620 FØR N = 2 TØ A2
1630 IF A(N) < A(N-1) THEN 1670
1640 IF B(N) < B(N-1) THEN 1670
1650 NEXT N
1660 IF B(A2) = 999 THEN 1690
1670 GØSUB 3660
1680 GØTØ 1560
1690 PRINT
1700 PRINT
1710 PRINT "TYPE IN THE SERVICE TIME INFØRMATIØN IN LIKE MANNER."
1720 FØR K = 1 TØ A3
1730 INPUT C(K), D(K)
1740 NEXT K
1750 PRINT
1760 FØR N = 2 TØ A3
1770 IF C(N) < C(N-1) THEN 1810
1780 IF D(N) < D(N-1) THEN 1810
1790 NEXT N
1800 IF D(A3) = 999 THEN 1830
1810 GØSUB 3660
1820 GØTØ 1710
1830 PRINT
1840 PRINT
1850 PRINT
1860 LET A7 = 10
1870 LET A8 = A7
1880 PRINT
1890 LET A9 = 1
1900 LET A1 = 0
1910 LET B1 = 0
1920 LET B5 = 0
1930 LET B2 = 0
1940 LET B9 = 0
1950 LET B3 = 0
1960 LET B4 = 1
1970 LET E(1) = 0
1980 LET F(1) = 0
1990 LET G(1) = 0
2000 LET H(1) = 0
2010 LET S(1) = 0
2020 LET T(1) = 0
2030 REM FIRST INTERARRIVAL TIME (J)
2040 LET L1 = RND(0)*1000
2050 FØR K = 1 TØ A2
```

```
2060 IF L1 <= B(K) THEN 2080
2070 GØTØ 2100
2080 LET L = K
2090 GØTØ2110
2100 NEXT K
2110 LET J(1)=A(L)
2120 LET I1 = 1
2130 LET J1 = 1
2140 LET K1 = 1
2150 REM SYSTEM ENTRANCE TIMES (E)
2160 LET E(I1+1)=J(I1)+E(I1)
2170 REM TEST FØR TIME ØVERFLØW
2180 IF E(I1+1) > A5 THEN 2610
2190 REM SUBSEQUENT INTERARRIVAL TIMES (J)
2200 LET L1 = RND(0)*1000
2210 FØR K = 1 TØ A2
2220 IF L1 <= B(K) THEN 2240
2230 GØTØ 2260
2240 LET L = K
2250 GØTØ 2270
2260 NEXT K
2270 LET J(I1+1)= A(L)
2280 LET K1 =K1+1
2290 REM QUEUE LENGTHS (G)
2300 LET G(K1)=G(K1-1)
2310 IF E(K1)<S(J1) THEN 2360
2320 IF E(K1)=S(J1) THEN 2340
2330 GØTØ 2380
2340 LET J1 = J1+1
2350 GØTØ 2430
2360 LET G(K1)=G(K1)+1
2370 GØTØ 2430
2380 LET J1 = J1+1
2390 IF G(K1) <= 0 THEN 2430
2400 LET G(K1)=G(K1)-1
2410 GØTØ 2310
2420 REM SERVICE TIMES (H)
2430 LET L1 = RND(0)*1000
2440 FØR K = 1 TØ A3
2450 IF L1>D(K) THEN 2480
2460 LET L = K
2470 GØTØ 2490
2480 NEXT K
2490 LET H(I1+1)=C(L)
2500 REM SERVICE ENTRANCE TIMES (T)
2510 IF S(I1)>E(I1+1) THEN 2540
2520 LET T(I1+1)=E(I1+1)
2530 GØTØ 2560
2540 LET T(I1+1)=S(I1)
2550 REM SERVICE DEPARTURE TIMES (S)
2560 LET S(I1+1)=H(I1+1)+T(I1+1)
2570 REM WAITING TIMES FØR SERVICE (F)
2580 LET F(I1+1)=T(I1+1)-E(I1+1)
```

```
2590 LET I1 =I1+1
2600 GØTØ 2160
2610 LET B6 = 0
2620 LET B7 = 0
2630 FØR K =2 TØ I1
2640 LET B6 = B6 + H(K)
2650 LET B7 = B7 + F(K)
2660 NEXT K
2670 REM AVERAGE SERVICE TIME
2680 LET B9 = B9 + B6 / (I1-1)
2690 LET B8 = I1 - 1
2700 REM AVERAGE WAITING TIME
2710 LET B1 = B1 + B7 / B8
2720 REM CUMULATIVE ARRIVALS
2730 LET B5 = B5 + B8
2740 LET B6 = 0
2750 REM AVERAGE QUEUE LENGTH
2760 FØR W = 1 TØ I1
2770 IF G(W) = 0 THEN 2890
2780 LET B7 = G(W)
2790 IF E(W+1) > S(W-B7) THEN 2820
2800 LET B6 = B6 + G(W) * J(W)
2810 GØTØ 2890
2820 LET B6 = B6 + G(W) * (S(W-B7) - E(W))
2830 FØR W1 = G(W) - 1 TØ 0 STEP -1
2840 IF E(W+1) >= S(W-W1) THEN 2870
2850 LET B6 = B6 + W1 * (E(W+1) - S(W-W1-1))
2860 GØTØ 2890
2870 LET B6 = B6 + W1 * (S(W-W1) - S(W-W1-1))
2880 NEXT W1
2890 NEXT W
2900 LET B2 = B2 + B6 / E(I1+1)
2910 REM IDLE TIMES (U)
2920 LET B6 = 0
2930 LET U(1)=J(1)
2940 LET B6 = B6 + U(1)
2950 FØR K = 2 TØ I1
2960 LET U(K) = E(K+1) - S(K)
2970 IF U(K) >= 0 THEN 2990
2980 LET U(K) = 0
2990 LET B6 = B6 + U(K)
3000 NEXT K
3010 REM AVERAGE FACILITY UTILIZATIØN
3020 LET B7 = B6
3030 LET B8 = E(I1+1)
3040 LET B3 =B3+1.-B7/B8
3050 LET A1 = A1 + 1
3060 LET B4 = B4+1
3070 REM TEST FØR SET CØMPLETED
3080 IF B4 <= A7 THEN 1970
3090 REM SET SUMMARIES
3100 LET M(A9) = B1
3110 LET V(A9) = B5
```

```
3120 LET N(A9) = B2
3130 LET W(A9) = B9
3140 LET Ø(A9) = B3
3150 REM FIRST SET ØNLY
3160 IF A9>1 THEN 3240
3170 LET P(1) = M(1) / A7
3180 LET X(1) = V(1) / A7
3190 LET Q(1) = N(1) / A7
3200 LET Y(1) = W(1) / A7
3210 LET R(1) = Ø(1) / A7
3220 GØTØ 3290
3230 REM ALL ØTHER SETS
3240 LET P(A9) = (P(A9-1) * (A8-A7)+M(A9)) / A8
3250 LET X(A9) = (X(A9-1) * (A8-A7)+V(A9)) / A8
3260 LET Q(A9) = (Q(A9-1) * (A8-A7)+N(A9)) / A8
3270 LET Y(A9) = (Y(A9-1) * (A8-A7)+W(A9)) / A8
3280 LET R(A9) = (R(A9-1) * (A8-A7)+Ø(A9)) / A8
3290 LET A8 = A8 + A7
3300 LET A9 = A9+1
3310 IF A9 <= A4 THEN 1910
3320 REM SUMMARY ØUTPUT SECTIØN
3330 PRINT
3340 PRINT "           S I M U L A T I Ø N   A V E R A G E S"
3350 PRINT "          ****************************************"
3360 PRINT
3370 PRINT
3380 PRINT "                                   THIS SET";
3390 PRINT "          CUMULATIVE"
3400 PRINT "                                   **** ***";
3410 PRINT "          **********"
3420 PRINT
3430 FØR K = 1 TØ A4
3440 PRINT
3450 PRINT "............"
3460 PRINT "AFTER"
3470 PRINT (K * A7)
3480 PRINT "SIMULATIØNS"
3490 PRINT "............"
3500 PRINT
3510 PRINT
3520 PRINT "NUMBER ØF ARRIVALS          "; (V(K) / A7), X(K)
3530 PRINT
3540 PRINT "WAITING TIME                "; (M(K) / A7), P(K)
3550 PRINT
3560 PRINT "QUEUE LENGTH                "; (N(K) / A7), Q(K)
3570 PRINT
3580 PRINT "SERVICE TIME                "; (W(K) / A7), Y(K)
3590 PRINT
3600 PRINT "FACILITY UTILIZATIØN        "; (Ø(K) / A7), R(K)
3610 PRINT
3620 PRINT
3630 NEXT K
3640 GØTØ 3700
```

```
3650 REM INPUT ERRØR SUBRØUTINE
3660 PRINT
3670 PRINT "UNACCEPTABLE INPUT!!!  CHECK INSTRUCTIØNS AND TRY AGAIN."
3680 PRINT
3690 RETURN
3700 STØP
3710 END
```

```
1000 REM ***** LP PRØGRAM - BASIC
1010 DIM A(7,12)
1020 PRINT
1030 PRINT
1040 PRINT "LINEAR PRØGRAMMING"
1050 PRINT "****** ***********"
1060 PRINT
1070 PRINT
1080 PRINT "SUPPLY THE FØLLØWING INFØRMATIØN BY ENTERING FIVE INTEGER"
1090 PRINT "NUMBERS ØN ØNE LINE, SEPARATED BY CØMMAS."
1100 PRINT
1110 PRINT "    (1) TYPE: MAX(1) ØR MIN(2)"
1120 PRINT "        NØ. ØF:"
1130 PRINT "    (2)     REAL VARIABLES"
1140 PRINT "    (3)     =< CØNSTRAINTS"
1150 PRINT "    (4)     => CØNSTRAINTS"
1160 PRINT "    (5)     =  CØNSTRAINTS"
1170 INPUT I1, I3, I4, I6, I7
1180 LET I2 = I4 + I6 + I7
1190 PRINT
1200 PRINT
1210 PRINT "CØNSTRAINT INPUT"
1220 PRINT ".......... ....."
1230 PRINT
1240 PRINT "THE RIGHTHAND SIDE AND CØEFFICIENT VALUES WILL BE"
1250 PRINT "REQUESTED FØR EACH CØNSTRAINT.  TØ ENTER THE CØEFFICIENT"
1260 PRINT "VALUES, TYPE THE VARIABLE NØ. AND THE CØEFF. VALUE,"
1270 PRINT "SEPARATED BY A CØMMA.  ZERØ CØEFF. NEED NØT BE ENTERED."
1280 PRINT "EXIT TØ NEXT CØNSTRAINT BY ENTERING 0,0"
1290 PRINT
1300 FØR I = 1 TØ I2
1310 IF I <= I4 THEN 1350
1320 IF I > I4 + I6 THEN 1370
1330 PRINT "CØNSTRAINT NØ. "; I; "   (=> TYPE)"
1340 GØ TØ 1380
1350 PRINT "CØNSTRAINT NØ. "; I; "   (=< TYPE)"
1360 GØ TØ 1380
1370 PRINT "CØNSTRAINT NØ. "; I; "   (=  TYPE)"
1380 PRINT
1390 PRINT "RIGHTHAND SIDE"
1400 INPUT A(I,3)
1410 PRINT "CØEFFICIENTS"
1420 FØR J = 1 TØ I3
1430 INPUT K, Z9
1440 IF K = 0 THEN 1470
1450 LET A(I,K+3) = Z9
1460 NEXT J
1470 NEXT I
1480 PRINT
1490 PRINT
1500 PRINT "ØBJECTIVE FUNCTIØN INPUT"
1510 PRINT ".......... ........ ....."
1520 PRINT
```

```
1530 PRINT "ENTER THE VARIABLE NUMBER AND THE COEFFICIENT VALUE IN"
1540 PRINT "A SIMILAR MANNER."
1550 PRINT
1560 LET J9 = I2 + I1
1570 FOR J = 1 TO I3
1580 INPUT K, A(J9,K+3)
1590 NEXT J
1600 SCRATCH:LPDAT
1610 MAT PRINT:LPDAT:A
1620 RESTORE:LPDAT:
1630 LET I8 = I3 + I4 + 2 * I6 + I7
1640 LET I9 = I6 + I7
1650 LET J1 = I3 + 3
1660 LET J2 = J1 + 1
1670 LET J3 = 0
1680 LET J4 = 0
1690 LET J5 = I8 + 3
1700 LET J6 = I2 + 3
1710 LET J7 = 0
1720 LET J8 = 0
1730 LET K1 = 0
1740 IF I1 = 2 THEN 1780
1750 LET K2 = I2 + 2
1760 LET A1 = 1.
1770 GO TO 1800
1780 LET K2 = I2 + 1
1790 LET A1 = -1.
1800 LET K = 0
1810 FOR I = 1 TO I2
1820 LET A(I,2) = 0.
1830 LET K = K + 1
1840 LET K3 = J1 + K
1850 IF I > I4 THEN 1940
1860 LET A(I,1) = K3 - 3
1870 FOR K4 = J2 TO J5
1880 IF K3 = K4 THEN 1910
1890 LET A(I,K4) = 0.
1900 GO TO 1920
1910 LET A(I,K4) = 1.
1920 NEXT K4
1930 GO TO 2170
1940 IF I > (I4 + I6) THEN 2060
1950 FOR K4 = J2 TO J5
1960 IF K3 <> K4 THEN 2020
1970 LET A(I,K4) = -1.
1980 LET J3 = K4 + I6
1990 LET A(I,J3) = 1.
2000 LET A(I,1) = J3 - 3
2010 GO TO 2040
2020 IF J3 = K4 THEN 2040
2030 LET A(I,K4) = 0.
2040 NEXT K4
2050 GO TO 2170
```

```
2060 IF I6 <= 0 THEN 2090
2070 LET J3 = J3 + 1
2080 GØ TØ 2100
2090 LET J3 = K3
2100 LET A(I,1) = J3 - 3
2110 FØR K4 = J2 TØ J5
2120 IF J3 = K4 THEN 2150
2130 LET A(I,K4) = 0.
2140 GØ TØ 2160
2150 LET A(I,K4) = 1.
2160 NEXT K4
2170 NEXT I
2180 LET K = J1 + I4 + I6
2190 FØR J = J2 TØ K
2200 LET A(J9,J) = 0.
2210 NEXT J
2220 LET K4 = K + I6 + I7
2230 IF K4 <= K THEN 2280
2240 LET K3 = K + 1
2250 FØR J = K3 TØ K4
2260 LET A(J9,J) = 9.9E50 * A1 * (-1.)
2270 NEXT J
2280 FØR I = 1 TØ I2
2290 LET K5 = A(I,1) + 3.
2300 LET A(I,2) = A(J9,K5)
2310 NEXT I
2320 FØR I = 1 TØ I2
2330 LET A(I2+1,3) = A(I2+1,3) + A(I,2) * A(I,3)
2340 NEXT I
2350 FØR J = 4 TØ J5
2360 LET A(K2,J) = 0.
2370 FØR I = 1 TØ I2
2380 LET A(K2,J) = A(K2,J) + A(I,2) * A(I,J)
2390 NEXT I
2400 LET A(J6,J) = A(I2+1,J) - A(I2+2,J)
2410 NEXT J
2420 LET K1 = K1 + 1
2430 FØR K6 = 4 TØ J5
2440 IF A(J6,K6) > 0. THEN 3070
2450 NEXT K6
2460 PRINT
2470 PRINT
2480 PRINT
2490 IF I1 = 2 THEN 2520
2500 PRINT "ØBJECT FUNCTIØN MAXIMIZED AT "; A(I2+1,3)
2510 GØ TØ 2530
2520 PRINT "ØBJECT FUNCTIØN MINIMIZED AT "; A(I2+1,3)
2530 PRINT
2540 PRINT "     ";K1;" ITERATIØNS REQUIRED"
2550 PRINT
2560 PRINT "INCLUDED VARIABLES"
2570 PRINT "........ ........."
2580 PRINT
```

```
2590 PRINT "NØ.        QUANTITY     TYPE"
2600 FØR K8 = 1 TØ I2
2610 LET Y9 = 9.9E50
2620 FØR I = 1 TØ I2
2630 IF Y9 < A(I,1) THEN 2660
2640 LET Y9 = A(I,1)
2650 LET K9 = I
2660 NEXT I
2670 LET L1 = Y9
2680 PRINT
2690 IF L1 <= I3 THEN 2730
2700 IF L1 > I3 + I4 THEN 2750
2710 PRINT L1; A(K9,3); "    SLACK"
2720 GØ TØ 2760
2730 PRINT L1; A(K9,3); "    REAL"
2740 GØ TØ 2760
2750 PRINT L1; A(K9,3); "    SURPLUS"
2760 LET A(K9,1) = 9.99E50
2770 NEXT K8
2780 PRINT
2790 PRINT "DØ YØU WISH TØ ALTER ANY DATA VALUES AND RUN AGAIN"
2800 PRINT "(YES = 1, NØ = 2)"
2810 INPUT N9
2820 IF N9 = 2 THEN 3560
2830 MAT INPUT:LPDAT:A
2840 PRINT
2850 PRINT "TØ ALTER A CØNSTRAINT CØEFF., ENTER THE CØNSTRAINT NØ.,"
2860 PRINT "VARIABLE NØ., AND NEW VALUE, SEPARATED BY CØMMAS"
2870 PRINT "(EXIT WITH 0,0,0)."
2880 INPUT I, K, Z9
2890 IF I = 0 THEN 2930
2900 IF K = 0 THEN 2930
2910 LET A(I,K+3) = Z9
2920 GØ TØ 2880
2930 PRINT
2940 PRINT "TØ ALTER A CØNSTRAINT RIGHTHAND SIDE VALUE, ENTER THE"
2950 PRINT "CØNSTRAINT NØ. AND THE NEW VALUE (EXIT WITH 0,0)"
2960 INPUT I, Z9
2970 IF I = 0 THEN 3000
2980 LET A(I,3) = Z9
2990 GØ TØ 2960
3000 PRINT
3010 PRINT "TØ ALTER AN ØBJECTIVE FUNCTIØN CØEFF., ENTER THE VARIABLE"
3020 PRINT "NØ. AND THE NEW VALUE (EXIT WITH 0,0)."
3030 INPUT K, Z9
3040 IF K = 0 THEN 1600
3050 LET A(J9,K+3) = Z9
3060 GØ TØ 3030
3070 IF K6 = J5 THEN 3140
3080 LET J = K6 + 1
3090 FØR K = J TØ J5
3100 IF A(J6,K) <= 0. THEN 3130
3110 IF A(J6,K6) >= A(J6,K) THEN 3130
```

```
3120 LET K6 = K
3130 NEXT K
3140 FØR K7 = 1 TØ I2
3150 IF A(K7,K6) > 0. THEN 3210
3160 NEXT K7
3170 PRINT
3180 PRINT
3190 PRINT "***** UNBØUNDED SØLUTIØN *****"
3200 GØ TØ 2780
3210 IF K7 = I2 THEN 3310
3220 LET A3 = A(K7,3) / A(K7,K6)
3230 LET J = K7 + 1
3240 FØR K = J TØ I2
3250 IF A(K,K6) <= 0. THEN 3300
3260 LET A4 = A(K,3) / A(K,K6)
3270 IF A3 <= A4 THEN 3300
3280 LET A3 = A4
3290 LET K7 = K
3300 NEXT K
3310 FØR I = 1 TØ I2
3320 IF I = K7 THEN 3370
3330 LET Q = A(I,K6) / A(K7,K6)
3340 FØR J = 3 TØ J5
3350 LET A(I,J) = A(I,J) - (A(K7,J) * Q)
3360 NEXT J
3370 NEXT I
3380 LET A(K7,1) = K6 - 3
3390 LET A(K7,2) = A(J9,K6)
3400 LET A5 = A(K7,K6)
3410 FØR J = 3 TØ J5
3420 LET A(K7,J) = A(K7,J) / A5
3430 NEXT J
3440 LET A(I2+1,3) = 0.
3450 FØR I = 1 TØ I2
3460 LET A(I2+1,3) = A(I2+1,3) + A(I,2) * A(I,3)
3470 NEXT I
3480 FØR J = 4 TØ J5
3490 LET A(K2,J) = 0.
3500 FØR I = 1 TØ I2
3510 LET A(K2,J) = A(K2,J) + A(I,2) * A(I,J)
3520 NEXT I
3530 LET A(J6,J) = A(I2+1,J) - A(I2+2,J)
3540 NEXT J
3550 GØ TØ 2420
3560 END
```

```
1000 REM ***** REGRESSIØN ANALYSIS--(REBAS)
1010 DIM A(40,6), B(6), C(6), D(6), E(6), F(6), G(6), H(6)
1020 DIM I(6), J(6), K(6), L(5), M(5,6), N(5), Ø(5), P(6)
1030 DIM Q(6), R(40,6), S(5), T(6), U(2), V(2), W(40), X(40)
1040 INPUT: RBDATA : I7, I6
1050 PRINT
1060 PRINT "THE SAMPLE SIZE (N) IS "; I6
1070 PRINT
1080 PRINT "INPUT THE TEST LIMITS FØR SIZE N"
1090 INPUT U(1), V(1)
1100 PRINT
1110 PRINT "INPUT THE TEST LIMITS FØR SIZE N-1"
1120 INPUT U(2), V(2)
1130 LET I2 = 0
1140 LET I4 = 0
1150 LET I5 = 1
1160 LET A3 = I6
1170 LET I9 = I7 - 1
1180 LET J1 = I9 + 1
1190 FØR I3 = 1 TØ I6
1200 FØR J2 = 1 TØ I7
1210 INPUT: RBDATA : A(I3,J2)
1220 NEXT J2
1230 NEXT I3
1240 GØSUB 5850
1250 PRINT "ØRIGINAL DATA"
1260 GØSUB 5860
1270 FØR I3 = 1 TØ I6
1280 FØR J2 = 1 TØ I7
1290 PRINT A(I3,J2),
1300 NEXT J2
1310 PRINT
1320 NEXT I3
1330 MAT R = (1) * A
1340 LET J3 = 0
1350 LET J4 = 1
1360 LET J5 = 0
1370 IF I4 <> 1 THEN 1400
1380 LET I6 = I6 - 1
1390 LET A3 = I6
1400 FØR J2 = 1 TØ I7
1410 LET B(J2) = 0
1420 FØR I3 = 1 TØ I6
1430 LET B(J2) = B(J2) + A(I3,J2)
1440 NEXT I3
1450 LET C(J2) = B(J2) / A3
1460 NEXT J2
1470 FØR J2 = 1 TØ I7
1480 LET D(J2) = 0
1490 FØR I3 = 1 TØ I6
1500 LET A(I3,J2) = A(I3,J2) - C(J2)
1510 LET D(J2) = D(J2) + A(I3,J2) ↑ 2
1520 NEXT I3
```

```
1530 LET E(J2) = SQR(D(J2) / (A3 - 1.))
1540 NEXT J2
1550 FØR J2 = 2 TØ I7
1560 LET F(J2) = 0
1570 FØR I3 = 1 TØ I6
1580 LET F(J2) = F(J2) + A(I3,1) * A(I3,J2)
1590 NEXT I3
1600 LET G(J2) = F(J2) / D(J2)
1610 LET H(J2) = C(1) - G(J2) * C(J2)
1620 NEXT J2
1630 IF I7 <> 2 THEN 1770
1640 FØR I3 = 1 TØ I6
1650 LET W(I3) = 0
1660 NEXT I3
1670 FØR I3 = 1 TØ I6
1680 FØR J2 = 1 TØ I7
1690 LET W(I3) = W(I3) + G(J2) * R(I3,J2)
1700 NEXT J2
1710 LET W(I3) = W(I3) + H(J2)
1720 NEXT I3
1730 GØSUB 5440
1740 IF I4 <> 1 THEN 1770
1750 IF J3 = 1 THEN 1350
1760 IF J3 = 2 THEN 2110
1770 FØR J2 = 2 TØ I7
1780 LET A6 = (D(1) - G(J2) * F(J2)) / (A3 - 2.)
1790 LET I(J2) = SQR(A6)
1800 LET J(J2) = SQR(A6 / D(J2))
1810 LET K(J2) = G(J2) / J(J2)
1820 LET L(J2) = 1. - (A6 / (D(1) / (A3 - 1.)))
1830 LET T(J2) = I(J2) / SQR(A3)
1840 NEXT J2
1850 GØSUB 5850
1860 PRINT "SIMPLE REGRESSIØN RESULTS"
1870 GØSUB 5860
1880 PRINT "VARIABLE",,"STANDARD"
1890 PRINT "NUMBER", "MEAN", "DEVIATIØN"
1900 PRINT
1910 FØR J2 = 1 TØ I7
1920 PRINT J2, C(J2), E(J2)
1930 NEXT J2
1940 PRINT
1950 PRINT "VARIABLE",,, "STD DEV", "STD DEV"
1960 PRINT "NUMBER", "A CØEFF", "B CØEFF", "REGRESS", "B"
1970 PRINT
1980 FØR J2 = 2 TØ I7
1990 PRINT J2, H(J2), G(J2), I(J2), J(J2)
2000 NEXT J2
2010 PRINT
2020 PRINT "VARIABLE",, "CØEFF ØF", "ERRØR ØF"
2030 PRINT "NUMBER", "T RATIØ", "DETERMIN", "ELEVATIØN"
2040 PRINT
2050 FØR J2 = 2 TØ I7
```

```
2060 PRINT J2, K(J2), L(J2), T(J2)
2070 NEXT J2
2080 IF I7 <> 2 THEN 2260
2090 IF J3 = 2 THEN 2110
2100 GØ TØ 4100
2110 LET I6 = I6 + 1
2120 LET A3 = I6
2130 FØR J2 = 1 TØ I7
2140 LET B(J2) = 0
2150 FØR I3 = 1 TØ I6
2160 LET B(J2) = B(J2) + R(I3,J2)
2170 NEXT I3
2180 LET C(J2) = B(J2) / A3
2190 NEXT J2
2200 FØR J2 = 1 TØ I7
2210 FØR I3 = 1 TØ I6
2220 LET A(I3,J2) = A(I3,J2) - C(J2)
2230 NEXT I3
2240 NEXT J2
2250 GØ TØ 4100
2260 FØR I3 = 2 TØ I7
2270 FØR J2 = 2 TØ I7
2280 LET M(I3-1,J2-1) = 0
2290 FØR J6 = 1 TØ I6
2300 LET M(I3-1,J2-1) = M(I3-1,J2-1) + A(J6,J2) * A(J6,I3)
2310 NEXT J6
2320 NEXT J2
2330 NEXT I3
2340 IF J4 = 2 THEN 2420
2350 FØR I3 = 2 TØ I7
2360 LET M(I3-1,I7) = 0
2370 FØR J6 = 1 TØ I6
2380 LET M(I3-1,I7) = M(I3-1,I7) + A(J6,1) * A(J6,I3)
2390 NEXT J6
2400 NEXT I3
2410 GØ TØ 2470
2420 LET J5 = J5 + 1
2430 FØR I3 = 2 TØ I7
2440 LET M(I3-1,I7) = 0
2450 NEXT I3
2460 LET M(J5,I7) = 1
2470 FØR I3 = 1 TØ I9
2480 LET Ø(I3) = I3
2490 LET N(I3) = I3
2500 NEXT I3
2510 LET J7 = I9 - 1
2520 FØR I3 = 1 TØ J7
2530 IF ABS(M(I3,I3)) > .001 THEN 2810
2540 FØR J8 = I3 TØ I9
2550 FØR J9 = I3 TØ I9
2560 IF ABS(M(J9,J8)) > .001 THEN 2630
2570 NEXT J9
2580 NEXT J8
```

```
2590 GØSUB 5850
2600 PRINT "GAUSS MULTIPLIERS CANNØT BE CØMPUTED"
2610 GØSUB 5860
2620 STØP
2630 IF J8 <= I3 THEN 2720
2640 LET N(J1) = N(I3)
2650 LET N(I3) = N(J8)
2660 LET N(J8) = N(J1)
2670 FØR K1 = 1 TØ I9
2680 LET M(K1,J1+1) = M(K1,I3)
2690 LET M(K1,I3) = M(K1,J8)
2700 LET M(K1,J8) = M(K1,J1+1)
2710 NEXT K1
2720 IF J9 <= I3 THEN 2810
2730 FØR K2 = I3 TØ J1
2740 LET M(I9+1,K2) = M(I3,K2)
2750 LET M(I3,K2) = M(J9,K2)
2760 LET M(J9,K2) = M(I9+1,K2)
2770 NEXT K2
2780 LET Ø(J1) = Ø(I3)
2790 LET Ø(I3) = Ø(J9)
2800 LET Ø(J9) = Ø(J1)
2810 LET A8 = M(I3,I3)
2820 LET K3 = I3 + 1
2830 FØR K4 = K3 TØ I9
2840 LET A9 = M(K4,I3)
2850 IF ABS(A9) <= .001 THEN 2890
2860 FØR J6 = I3 TØ J1
2870 LET M(K4,J6) = M(K4,J6) - (A9 * M(I3,J6) / A8)
2880 NEXT J6
2890 NEXT K4
2900 NEXT I3
2910 IF ABS(M(I9,I9)) > .0001 THEN 2960
2920 GØSUB 5850
2930 PRINT "GAUSS MULTIPLIERS CANNØT BE CØMPUTED"
2940 GØSUB 5860
2950 STØP
2960 LET M(I9,J1) = M(I9,J1) / M(I9,I9)
2970 LET K5 = J1
2980 FØR K4 = 1 TØ J7
2990 LET K5 = K5 - 1
3000 LET K3 = K5 - 1
3010 LET B1 = 0
3020 FØR I3 = K5 TØ I9
3030 LET B1 = B1 + M(K3,I3) * M(I3,J1)
3040 NEXT I3
3050 LET M(K3,J1) = (M(K3,J1) - B1) / M(K3,K3)
3060 NEXT K4
3070 FØR K5 = 1 TØ I9
3080 IF N(K5) = K5 THEN 3190
3090 FØR K4 = 1 TØ I9
3100 IF N(K4) <> K5 THEN 3180
3110 LET M(J1+1,J1) = M(K5,J1)
```

```
3120 LET M(K5,J1) = M(K4,J1)
3130 LET M(K4,J1) = M(J1+1,J1)
3140 LET N(J1) = N(K5)
3150 LET N(K5) = N(K4)
3160 LET N(K4) = N(J1)
3170 GØ TØ 3190
3180 NEXT K4
3190 NEXT K5
3200 IF I5 = 2 THEN 4500
3210 IF J4 > 1 THEN 3270
3220 FØR I3 = 1 TØ I9
3230 LET G(I3+1) = M(I3,J1)
3240 NEXT I3
3250 LET J4 = J4 + 1
3260 GØ TØ 2260
3270 LET Q(J5) = M(J5,J1)
3280 IF J5 < I9 THEN 2260
3290 LET G(1) = 0
3300 FØR I3 = 2 TØ I7
3310 LET G(1) = G(1) + G(I3) * C(I3)
3320 NEXT I3
3330 LET G(1) = C(1) - G(1)
3340 FØR I3 = 1 TØ I6
3350 LET W(I3) = 0
3360 NEXT I3
3370 FØR I3 = 1 TØ I6
3380 FØR J2 = 2 TØ I7
3390 LET W(I3) = W(I3) + G(J2) * R(I3,J2)
3400 NEXT J2
3410 LET W(I3) = W(I3) + G(1)
3420 NEXT I3
3430 GØSUB 5440
3440 IF I4 <> 1 THEN 3470
3450 IF J3 = 2 THEN 5430
3460 GØ TØ 1350
3470 LET B2 = I7
3480 LET B = 0
3490 FØR J2 = 2 TØ I7
3500 LET B3 = B3 + G(J2) * F(J2)
3510 NEXT J2
3520 LET A6 = (D(1) - B3) / (A3 - B2)
3530 LET B4 = SQR(A6)
3540 LET B5 = 1. - (A6 / (D(1) / (A3 - 1.)))
3550 FØR J2 = 2 TØ I7
3560 LET P(J2) = G(J2) * SQR(D(J2) / D(1))
3570 NEXT J2
3580 FØR J2 = 2 TØ I7
3590 LET J(J2) = SQR(Q(J2-1) * A6)
3600 LET K(J2) = G(J2) / J(J2)
3610 NEXT J2
3620 LET A8 = (B3 / (B2 - 1.)) / A6
3630 GØSUB 5850
3640 PRINT "MULTIPLE REGRESSIØN RESULTS"
```

```
3650 GØSUB 5860
3660 PRINT "VARIABLE", "NET REG", "STD DEV"
3670 PRINT "NUMBER", "CØEFF", "REGRESS", "T RATIØ", "B CØEFF"
3680 PRINT
3690 FØR J2 = 2 TØ I7
3700 PRINT J2, G(J2), J(J2), K(J2), P(J2)
3710 NEXT J2
3720 PRINT
3730 PRINT "A = "; G(1)
3740 PRINT "RESID VAR = "; B4
3750 PRINT "R2 = "; B5
3760 PRINT "F = "; A8
3770 GØSUB 5850
3780 PRINT "TEST FØRECASTS"
3790 GØSUB 5860
3800 PRINT "ARE TEST FØRECASTS DESIRED, (YES = 1, NØ = 0)"
3810 INPUT K9
3820 IF K9 <= 0 THEN 4100
3830 PRINT
3840 PRINT "ENTER THE X TEST VALUES IN ØRDER, ØNE PER LINE."
3850 PRINT "EXIT WITH ALL ZERØS."
3860 FØR J2 = 2 TØ I7
3870 INPUT S(J2)
3880 NEXT J2
3890
3900 FØR J2 = 2 TØ I7
3910 IF S(J2) <> 0 THEN 3940
3920 NEXT J2
3930 GØ TØ 4100
3940 LET B6 = I6
3950 LET B7 = G(1)
3960 FØR J2 = 2 TØ I7
3970 LET B7 = B7 + G(J2) * S(J2)
3980 NEXT J2
3990 PRINT
4000 PRINT "FØRECAST VALUE IS "; B7
4010 IF J3 >= 2 THEN 4090
4020 LET B8 = 0
4030 FØR J2 = 2 TØ I7
4040 LET B8 = B8 + SQR(Q(J2-1)) * (S(J2) - C(J2))
4050 NEXT J2
4060 LET B9 = SQR(A6 * (1. + 1. / B6 + B8 ↑ 2))
4070 PRINT "STANDARD ERRØR ØF THIS FØRECAST IS "; B9
4080 PRINT
4090 GØ TØ 3860
4100 GØSUB 5850
4110 PRINT "SIMPLE PARABØLIC REGRESSIØNS"
4120 GØSUB 5860
4130 LET I9 = 2
4140 LET J1 = 3
4150 LET C1 = 3
4160 LET I5 = 2
4170 FØR J2 = 2 TØ I7
```

```
4180 LET J4 = 1
4190 LET J5 = 0
4200 LET B3 = 0
4210 FØR I3 = 1 TØ I6
4220 LET A(I3,I7+1) = (A(I3,J2) + C(J2)) ↑ 2
4230 LET B3 = B3 + A(I3,I7+1)
4240 NEXT I3
4250 LET C(I7+1) = B3 / A3
4260 LET D(I7+1) = 0
4270 LET F(I7+1) = 0
4280 FØR I3 = 1 TØ I6
4290 LET A(I3,I7+1) = A(I3,I7+1) - C(I7+1)
4300 LET D(I7+1) = D(I7+1) + A(I3,I7+1) ↑ 2
4310 LET F(I7+1) = F(I7+1) + A(I3,1) * A(I3,I7+1)
4320 NEXT I3
4330 LET B3 = 0
4340 FØR J6 = 1 TØ I6
4350 LET B3 = B3 + A(J6,J2) * A(J6,I7+1)
4360 NEXT J6
4370 LET M(1,1) = D(J2)
4380 LET M(1,2) = B3
4390 LET M(2,1) = B3
4400 LET M(2,2) = D(I7+1)
4410 IF J4 = 2 THEN 4450
4420 LET M(1,3) = F(J2)
4430 LET M(2,3) = F(I7+1)
4440 GØ TØ 2470
4450 LET J5 = J5 + 1
4460 LET M(1,3) = 0
4470 LET M(2,3) = 0
4480 LET M(J5,3) = 1
4490 GØ TØ 2470
4500 IF J4 > 1 THEN 4550
4510 LET G(2) = M(1,J1)
4520 LET G(3) = M(2,J1)
4530 LET J4 = J4 + 1
4540 GØ TØ 4370
4550 LET Q(J5) = M(J5,J1)
4560 IF J5 < I9 THEN 4370
4570 LET G(1) = C(1) - G(2) * C(J2) - G(3) * C(I7+1)
4580 LET B3 = G(2) * F(J2) + G(3) * F(I7+1)
4590 LET A6 = (D(1) - B3) / (A3 - C1)
4600 LET B4 = SQR(A6)
4610 LET B5 = 1. - (A6 / (D(1) / (A3 - 1.)))
4620 LET P(2) = G(2) * SQR(D(J2) / D(1))
4630 LET P(3) = G(3) * SQR(D(I7+1) / D(1))
4640 LET J(2) = SQR(Q(1) * A6)
4650 LET J(3) = SQR(Q(2) * A6)
4660 LET K(2) = G(2) / J(2)
4670 LET K(3) = G(3) / J(3)
4680 LET A8 = (B3 / 2.) / A6
4690 LET R(1,J2) = G(1)
4700 FØR J6 = 2 TØ 3
```

```
4710 LET R(J6,J2) = G(J6)
4720 LET R(J6+2,J2) = J(J6)
4730 LET R(J6+4,J2) = K(J6)
4740 LET R(J6+6,J2) = P(J6)
4750 NEXT J6
4760 LET R(10,J2) = B4
4770 LET R(11,J2) = B3
4780 LET R(12,J2) = B5
4790 LET R(13,J2) = A8
4800 NEXT J2
4810 PRINT "VARIABLE"
4820 PRINT "NUMBER", "A", "B1", "B2"
4830 PRINT
4840 FØR J2 = 2 TØ I7
4850 PRINT J2, R(1,J2), R(2,J2), R(3,J2)
4860 NEXT J2
4870 PRINT
4880 PRINT "VARIABLE", "STD DEV", "STD DEV"
4890 PRINT "NUMBER", "B1", "B2", "T1", "T2"
4900 PRINT
4910 FØR J2 = 2 TØ I7
4920 PRINT J2, R(4,J2), R(5,J2), R(6,J2), R(7,J2)
4930 NEXT J2
4940 PRINT
4950 PRINT "VARIABLE",,, "STD DEV"
4960 PRINT "NUMBER", "BETA 1", "BETA 2", "REGRESS"
4970 PRINT
4980 FØR J2 = 2 TØ I7
4990 PRINT J2, R(8,J2), R(9,J2), R(10,J2)
5000 NEXT J2
5010 PRINT
5020 PRINT "VARIABLE", "EXP", "MULT", "F"
5030 PRINT "NUMBER", "VAR", "R2", "RATIØ"
5040 PRINT
5050 FØR J2 = 2 TØ I7
5060 PRINT J2, R(11,J2), R(12,J2), R(13,J2)
5070 NEXT J2
5080 IF I7 = 2 THEN 5430
5090 GØSUB 5850
5100 PRINT "SIMPLE LINEAR INTERCØRRELATIØNS"
5110 GØSUB 5860
5120 LET K6 = I7 - 1
5130 FØR J2 = 2 TØ K6
5140 FØR I3 = 1 TØ I6
5150 LET A(I3,1) = A(I3,J2)
5160 NEXT I3
5170 LET C(1) = C(J2)
5180 LET D(1) = D(J2)
5190 LET K7 = J2 + 1
5200 FØR J8 = K7 TØ I7
5210 LET F(J8) = 0
5220 FØR I3 = 1 TØ I6
5230 LET F(J8) = F(J8) + A(I3,1) * A(I3,J8)
```

```
5240 NEXT I3
5250 LET G(J8) = F(J8) / D(J8)
5260 LET H(J8) = C(1) - G(J8) * C(J8)
5270 LET A6 = (D(1) - G(J8) * F(J8)) / (A3 - 2.)
5280 LET I(J8) = SQR(A6)
5290 LET J(J8) = SQR(A6 / D(J8))
5300 LET K(J8) = G(J8) / J(J8)
5310 LET L(J8) = 1. - (A6 / (D(J2) / (A3 - 1.)))
5320 PRINT " DV IV"
5330 PRINT J2; J8
5340 PRINT "          A = "; H(J8)
5350 PRINT "          B = "; G(J8)
5360 PRINT "          STD ERR ØF EST = "; I(J8)
5370 PRINT "          STD DEV ØF B = "; J(J8)
5380 PRINT "          T RATIØ = "; K(J8)
5390 PRINT "          R2 = "; L(J8)
5400 PRINT
5410 NEXT J8
5420 NEXT J2
5430 STØP
5440 LET I4 = 0
5450 LET J3 = J3 + 1
5460 FØR I3 = 1 TØ I6
5470 LET X(I3) = R(I3,1) - W(I3)
5480 NEXT I3
5490 LET K8 = I6 - 1
5500 LET C2 = 0
5510 FØR I3 = 1 TØ K8
5520 LET C2 = C2 + (X(I3+1) - X(I3)) ↑ 2
5530 NEXT I3
5540 LET C3 = 0
5550 FØR I3 = 1 TØ I6
5560 LET C3 = C3 + X(I3) ↑ 2
5570 NEXT I3
5580 LET B6 = I6
5590 LET C4 = K8
5600 LET C5 = (C2 / C4) / (C3 / B6)
5610 IF U(J3) > C5 THEN 5640
5620 IF C5 > V(J3) THEN 5640
5630 GØ TØ 5840
5640 LET I4 = 1
5650 IF J3 >= 2 THEN 5710
5660 GØSUB 5850
5670 PRINT "AUTØCØRRELATIØN EXISTS"
5680 PRINT "K = "; U(J3); "   V = "; C5; "   KPRIME = "; V(J3)
5690 GØSUB 5860
5700 GØ TØ 5760
5710 GØSUB 5850
5720 PRINT "AUTØCØRRELATIØN EXISTS WHEN FIRST DIFFS"
5730 PRINT "ARE USED AS VARIABLES--STØP"
5740 GØSUB 5860
5750 GØ TØ 5840
5760 FØR J2 = 1 TØ I7
```

```
5770 FØR I3 = 1 TØ K8
5780 LET A(I3,J2) = R(I3+1,J2) - R(I3,J2)
5790 NEXT I3
5800 NEXT J2
5810 GØSUB 5850
5820 PRINT "REGRESSIØN CØEFFS CALCULATED FRØM FIRST DIFFS"
5830 GØSUB 5860
5840 RETURN
5850 PRINT
5860 PRINT
5870 PRINT
5880 RETURN
5890 END
```

```
                 Q U E I N G   S I M U L A T I Ø N
                 <><><><><><><><><><><><><><><><><><><>


     I N P U T   D A T A
     ******************

HØW MANY INTERARRIVAL TIME PØSSIBILITIES
(1 TØ 10)
? 5


HØW MANY SERVICE TIME PØSSIBILITIES (1 TØ 10)
? 4


HØW MANY SETS ØF 100 SIMULITIØNS DESIRED
(1 TØ 10)
? 10


DESIRED TIME PERIØD IN MINUTES
? 100


IN ASCENDING ØRDER, TYPE IN EACH INTERARRIVAL
TIME PØSSIBILITY AND THE UPPER BØUND ØF THE
ASSØCIATED RANDØM NUMBER SET ØN A LINE.
? 2,99

? 4,299

? 6,699

? 8,899

? 10,999


TYPE IN THE SERVICE TIME INFØRMATIØN IN LIKE
MANNER.
? 2,399

? 4,699

? 6,899

? 8,999
```

SIMULATIØN AVERAGES

	THIS SET	CUMULATIVE
AFTER 100 SIMULATIØNS		
NUMBER ØF ARRIVALS	16.40000	16.40000
WAITING TIME	0.87715	0.87715
QUEUE LENGTH	0.14022	0.14022
SERVICE TIME	4.11213	4.11213
FACILITY UTILIZATIØN	0.63380	0.63380

	THIS SET	CUMULATIVE
AFTER 1000 SIMULATIØNS		
NUMBER ØF ARRIVALS	16.80000	16.45000
WAITING TIME	1.49939	0.85619
QUEUE LENGTH	0.24911	0.13781
SERVICE TIME	4.28783	4.00890
FACILITY UTILIZATIØN	0.67577	0.62253

```
LINEAR PRØGRAMMING
****** ***********


SUPPLY THE FØLLØWING INFØRMATIØN BY ENTERING
FIVE INTEGER NUMBERS ØN ØNE LINE, SEPARATED
BY CØMMAS.

    (1) TYPE: MAX(1) ØR MIN(2)
        NØ. ØF:
    (2)    REAL VARIABLES
    (3)     =< CØNSTRAINTS
    (4)     => CØNSTRAINTS
    (5)     =  CØNSTRAINTS
? 2,15,3,5,0


CØNSTRAINT INPUT
.......... .....

THE RIGHTHAND SIDE AND CØEFFICIENT VALUES WILL
BE REQUESTED FØR EACH CØNSTRAINT. TØ INPUT THE
CØEFFICIENT VALUES, ENTER THE VARIABLE NØ. AND
THE CØEFF. VALUE, SEPARATED BY A CØMMA. ZERØ
CØEFF. NEED NØT BE ENTERED. EXIT TØ NEXT
CØNSTRAINT BY ENTERING 0,0

CØNSTRAINT NØ. 1    (=< TYPE)

RIGHTHAND SIDE
? 200

CØEFFICIENTS
? 1,1

? 2,1

? 3,1

? 4,1

? 5,1

? 0,0
```

CØNSTRAINT NØ. 2 (=< TYPE)

RIGHTHAND SIDE
? 100

CØEFFICIENTS
? 6,1

? 7,1

? 8,1

? 9,1

? 10,1

? 0,0

CØNSTRAINT NØ. 3 (=< TYPE)

RIGHTHAND SIDE
? 150

CØEFFICIENTS
? 11,1

? 12,1

? 13,1

? 14,1

? 15,1

? 0,0

CØNSTRAINT NØ. 4 (=> TYPE)

RIGHTHAND SIDE
? 80

CØEFFICIENTS
? 1,1

? 6,1

? 11,1

? 0,0

```
CØNSTRAINT NØ. 5    (=> TYPE)

RIGHTHAND SIDE
? 90

CØEFFICIENTS
? 2,1

? 7,1

? 12,1

? 0,0

CØNSTRAINT NØ. 6    (=> TYPE)

RIGHTHAND SIDE
? 100

CØEFFICIENTS
? 3,1

? 8,1

? 13,1

? 0,0

CØNSTRAINT NØ. 7    (=> TYPE)

RIGHTHAND SIDE
? 70

CØEFFICIENTS
? 4,1

? 9,1

? 14,1

? 0,0
```

```
CØNSTRAINT NØ. 8    (=> TYPE)

RIGHTHAND SIDE
? 60

CØEFFICIENTS
? 5,1

? 10,1

? 15,1

? 0,0


ØBJECTIVE FUNCTIØN INPUT
.........................

ENTER THE VARIABLE NUMBER AND THE CØEFFICIENT
VALUE IN A SIMILAR MANNER.

? 1,5

? 2,1

? 3,6

? 4,3

? 5,1

? 6,2

? 7,3

? 8,4

? 9,
? 9,5

? 10,4

? 11,4

? 12,2

? 13,3

? 14,2

? 15,3
```

```
ØBJECT FUNCTIØN MINIMIZED AT    7.70000E+02

      16 ITERATIØNS REQUIRED

INCLUDED VARIABLES
........ .........

NØ.        QUANTITY       TYPE

 2        9.00000E+01     REAL

 4        0.00000E+00     REAL

 5        6.00000E+01     REAL

 6        8.00000E+01     REAL

 8        2.00000E+01     REAL

13        8.00000E+01     REAL

14        7.00000E+01     REAL

16        5.00000E+01     SLACK


DØ YØU WISH TØ ALTER ANY DATA VALUES AND
RUN AGAIN (YES = 1, NØ = 2)
? 1


TØ ALTER A CØNSTRAINT CØEFF., ENTER THE
CØNSTRAINT NØ., VARIABLE NØ., AND NEW VALUE,
SEPARATED BY CØMMAS (EXIT WITH 0,0,0).
? 0,0,0


TØ ALTER A CØNSTRAINT RIGHTHAND SIDE VALUE,
ENTER THE CØNSTRAINT NØ. AND THE NEW VALUE
(EXIT WITH 0,0).
? 0,0


TØ ALTER AN ØBJECTIVE FUNCTIØN CØEFF., ENTER
THE VARIABLE NØ. AND NEW VALUE (EXIT WITH 0,0).
? 3,1

? 0,0
```

```
ØBJECT FUNCTIØN MINIMIZED AT    6.00000E+02

      15 ITERATIØNS REQUIRED

INCLUDED VARIABLES
.....................

NØ.          QUANTITY         TYPE

 2          4.00000E+01      REAL
 3          1.00000E+02      REAL
 5          6.00000E+01      REAL
 6          8.00000E+01      REAL
12          5.00000E+01      REAL
14          7.00000E+01      REAL
17          2.00000E+01      SLACK
18          3.00000E+01      SLACK

DØ YØU WISH TØ ALTER ANY DATA VALUES AND
RUN AGAIN (YES = 1, NØ = 2)
? 2
```

THE SAMPLE SIZE (N) IS 20

INPUT THE TEST LIMITS FØR SIZE N
? 1.37,2.84

INPUT THE TEST LIMITS FØR SIZE N-1
? 1.35,2.87

ØRIGINAL DATA

0.960000E+02	0.200000E+01	0.180000E+02	0.600000E+01
0.830000E+02	0.300000E+01	0.000000E+00	0.220000E+02
0.126000E+03	0.400000E+01	0.140000E+02	0.180000E+02
0.610000E+02	0.100000E+01	0.600000E+01	0.800000E+01
0.590000E+02	0.100000E+01	0.100000E+01	0.120000E+02
0.900000E+02	0.100000E+01	0.900000E+01	0.100000E+02
0.820000E+02	0.300000E+01	0.600000E+01	0.170000E+02
0.880000E+02	0.200000E+01	0.120000E+02	0.110000E+02
0.860000E+02	0.200000E+01	0.700000E+01	0.160000E+02
0.760000E+02	0.300000E+01	0.200000E+01	0.230000E+02
0.102000E+03	0.200000E+01	0.170000E+02	0.700000E+01
0.108000E+03	0.300000E+01	0.150000E+02	0.120000E+02
0.960000E+02	0.400000E+01	0.700000E+01	0.240000E+02
0.700000E+02	0.200000E+01	0.000000E+00	0.160000E+02
0.800000E+02	0.100000E+01	0.120000E+02	0.900000E+01
0.113000E+03	0.300000E+01	0.160000E+02	0.110000E+02
0.760000E+02	0.200000E+01	0.200000E+01	0.220000E+02
0.740000E+02	0.100000E+01	0.600000E+01	0.110000E+02
0.980000E+02	0.200000E+01	0.120000E+02	0.160000E+02
0.800000E+02	0.200000E+01	0.150000E+02	0.800000E+01

SIMPLE REGRESSIØN RESULTS

VARIABLE NUMBER	MEAN	STANDARD DEVIATIØN
1	0.872000E+02	0.169445E+02
2	0.220000E+01	0.951453E+00
3	0.885000E+01	0.596724E+01
4	0.139500E+02	0.565197E+01

VARIABLE NUMBER	A CØEFF	B CØEFF	STD DEV REGRESS	STD DEV B
2	0.624884E+02	0.112325E+02	0.135094E+02	0.325741E+01
3	0.694019E+02	0.201109E+01	0.122902E+02	0.472511E+00
4	0.868736E+02	0.233957E-01	0.174083E+02	0.706609E+00

VARIABLE NUMBER	T RATIØ	CØEFF ØF DETERMIN	ERRØR ØF ELEVATIØN
2	0.344831E+01	0.364354E+00	0.302079E+01
3	0.425617E+01	0.473902E+00	0.274819E+01
4	0.331098E-01	-.554913E-01	0.389261E+01

MULTIPLE REGRESSIØN RESULTS

VARIABLE NUMBER	NET REG CØEFF	STD DEV REGRESS	T RATIØ	B CØEFF
2	0.502990E+01	0.371819E+01	0.135278E+01	0.282435E+00
3	0.262735E+01	0.590808E+00	0.444704E+01	0.925256E+00
4	0.120583E+01	0.789248E+00	0.152782E+01	0.402214E+00

A = 0.360609E+02
RESID VAR = 0.746464E+01
R2 = 0.806
F = 27.30

TEST FØRECASTS

ARE TEST FØRECASTS DESIRED (YES = 1, NØ = 0)
? 1

ØN ØNE LINE, ENTER THE X TEST VALUES IN ØRDER,
SEPARATED BY CØMMAS. EXIT WITH ALL ZERØS.
? 50,1.5,10

FØRECAST VALUE IS 0.303555E+03
STANDARD ERRØR ØF THIS FØRECAST IS 0.170441E+03

? 0,0,0

SIMPLE PARABØLIC REGRESSIØNS

VARIABLE NUMBER	A	B1	B2
2	0.639103E+02	0.980000E+01	0.303448E+00
3	0.701469E+02	0.169959E+01	0.179379E-01
4	0.804826E+02	0.100703E+01	-.325886E-01

VARIABLE NUMBER	STD DEV B1	STD DEV B2	T1	T2
2	0.163262E+02	0.338462E+01	0.600263E+00	0.896549E-01
3	0.182130E+01	0.101082E+00	0.933176E+00	0.177458E+00
4	0.461005E+01	0.150827E+00	0.218444E+00	-.216066E+00

VARIABLE NUMBER	BETA 1	BETA 2	STD DEV REGRESS
2	0.550282E+00	0.821898E-01	0.138978E+02
3	0.598534E+00	0.113820E+00	0.126349E+02
4	0.335906E+00	-.332249E+00	0.178884E+02

VARIABLE NUMBER	EXP VAR	MULT R2	F RATIØ
2	0.217168E+04	0.327282E+00	0.562181E+01
3	0.274131E+04	0.443985E+00	0.858588E+01
4	0.152711E+02	-.114518E+00	0.238614E-01

SIMPLE LINEAR INTERCØRRELATIØNS

```
DV    IV
 2     3
          A =    0.204826E+01
          B =    0.171458E-01
          STD ERR ØF EST =    0.971857E+00
          STD DEV ØF B =   0.373639E-01
          T RATIØ =    0.458887E+00
          R2 =    -.433497E-01

DV    IV
 2     4
          A =    0.747426E+00
          B =    0.104127E+00
          STD ERR ØF EST =    0.768084E+00
          STD DEV ØF B =   0.311769E-01
          T RATIØ =    0.333989E+01
          R2 =    0.348307E+00

DV    IV
 3     4
          A =    0.179091E+02
          B =    -.649395E+00
          STD ERR ØF EST =    0.483386E+01
          STD DEV ØF B =   0.196208E+00
          T RATIØ =    -.330972E+01
          R2 =    0.343792E+00
STØP
```

SYSTEM: Ø

DATE:

##	I	J	TIME	CØN	CPU	IØ	MISC
1	8	1	9:00				
2	1	1	9:20				
3	6	1	9:25				
4	7	1	9:30				
5	5	1	9:50				
6	1	2	9:55				
7	5	2	10:00				
8	2	1	10:05				
9	2	2	10:10				
10	6	2	10:15				
11	6	3	10:20				
12	5	3	10:25				
13	9	1	10:30				
14	8	2	10:50				
15	4	1	11:10				
16	7	2	11:20				
17	2	3	11:40				
18	8	3	11:45				
19	3	1	12:05				
20	1	3	12:10				
21	4	2	12:15				
22	3	2	12:25				
23	3	3	12:30				
24	4	3	12:35				
25	9	2	12:45				
26	7	3	1:05				
27	9	3	1:25				

SYSTEM A

TIMES AND CHARGES
****

		CONNECT		CPU		I/Ø		TØTAL
TASK	RUN	(HRS)	($)	(SEC)	($)	(SEC)	($)	CHARGE
1	1	.012	0.12	17.0	0.85	1.4	0.04	1.01
	2	.012	0.12	16.8	0.84	1.6	0.05	1.01
	3	.017	0.17	17.9	0.89	3.0	0.09	1.16
AVERAGE		.014	0.14	17.2	0.86	2.0	0.06	1.06
2	1	.060	0.60	18.5	0.93	7.8	0.23	1.76
	2	.057	0.57	19.4	0.97	7.0	0.21	1.75
	3	.052	0.52	18.2	0.91	5.7	0.17	1.61
AVERAGE		.056	0.56	18.7	0.93	6.8	0.20	1.70
3	1	.046	0.46	2.5	0.13	3.2	0.10	0.68
	2	.046	0.46	2.9	0.14	3.7	0.11	0.71
	3	.045	0.45	2.6	0.13	2.1	0.06	0.64
AVERAGE		.045	0.45	2.7	0.13	3.0	0.09	0.68
4	1	.096	0.96	4.8	0.24	39.9	1.20	2.40
	2	.139	1.39	5.3	0.26	37.7	1.13	2.78
	3	.132	1.32	5.4	0.27	37.4	1.12	2.71
AVERAGE		.122	1.22	5.2	0.26	38.3	1.15	2.63
5	1	.112	1.12	2.5	0.13	28.0	0.84	2.08
	2	.027	0.27	3.1	0.15	11.7	0.35	0.78
	3	.026	0.26	1.9	0.09	12.2	0.37	0.72
AVERAGE		.055	0.55	2.5	0.12	17.3	0.52	1.19
6	1	.023	0.23	18.9	0.94	5.5	0.16	1.34
	2	.024	0.24	19.6	0.98	5.2	0.16	1.38
	3	.023	0.23	19.3	0.97	3.6	0.11	1.31
AVERAGE		.024	0.24	19.3	0.96	4.8	0.14	1.34

SYSTEM A

TIMES AND CHARGES (CØNT.)
****

		CØNNECT		CPU		I/Ø		TØTAL
TASK	RUN	(HRS)	($)	(SEC)	($)	(SEC)	($)	CHARGE
7	1	.982	9.82	344.6	17.23	64.5	1.93	28.98
	2	.714	7.14	344.7	17.23	63.0	1.89	26.27
	3	.276	2.76	340.1	17.01	49.7	1.49	21.25
AVERAGE		.657	6.57	343.1	17.16	59.1	1.77	25.50
8	1	.278	2.78	82.1	4.10	120.4	3.61	10.50
	2	.364	3.64	81.1	4.06	119.2	3.58	11.27
	3	.308	3.08	82.3	4.11	112.8	3.38	10.58
AVERAGE		.316	3.16	81.8	4.09	117.5	3.52	10.78
9	1	.157	1.57	18.4	0.92	20.5	0.62	3.11
	2	.152	1.52	18.4	0.92	18.8	0.56	3.01
	3	.153	1.53	18.2	0.91	20.3	0.61	3.05
AVERAGE		.154	1.54	18.3	0.92	19.9	0.60	3.05
ØVERALL AVERAGE		.161	1.61	56.5	2.83	29.8	0.90	5.33

SYSTEM B

TIMES AND CHARGES
****

TASK	RUN	CØNNECT (HRS)	CØNNECT ($)	CPU (SEC)	CPU ($)	I/Ø (SEC)	I/Ø ($)	TØTAL CHARGE
1	1	.014	0.14	13.1	0.65	1.9	0.06	0.85
	2	.013	0.13	13.3	0.66	1.7	0.05	0.85
	3	.010	0.10	12.9	0.64	1.8	0.05	0.80
AVERAGE		.012	0.12	13.1	0.65	1.8	0.05	0.83
2	1	.059	0.59	15.8	0.79	6.7	0.20	1.58
	2	.053	0.53	15.4	0.77	4.8	0.14	1.44
	3	.071	0.71	15.8	0.79	5.5	0.16	1.66
AVERAGE		.061	0.61	15.7	0.78	5.7	0.17	1.56
3	1	.047	0.47	2.6	0.13	3.4	0.10	0.70
	2	.052	0.52	3.4	0.17	2.9	0.09	0.78
	3	.046	0.46	2.8	0.14	3.4	0.10	0.70
AVERAGE		.048	0.48	2.9	0.15	3.2	0.10	0.73
4	1	.105	1.05	5.9	0.29	26.6	0.80	2.14
	2	.195	1.95	6.4	0.32	37.8	1.13	3.40
	3	.086	0.86	6.2	0.31	23.3	0.70	1.87
AVERAGE		.129	1.29	6.2	0.31	29.2	0.88	2.47
5	1	.163	1.63	4.4	0.22	37.1	1.11	2.97
	2	.059	0.59	3.5	0.17	27.0	0.81	1.58
	3	.060	0.60	3.5	0.17	23.8	0.71	1.49
AVERAGE		.094	0.94	3.8	0.19	29.3	0.88	2.01
6	1	.023	0.23	17.1	0.86	3.6	0.11	1.20
	2	.030	0.30	17.2	0.86	3.9	0.12	1.28
	3	.020	0.20	17.9	0.89	2.4	0.07	1.17
AVERAGE		.025	0.25	17.4	0.87	3.3	0.10	1.21

SYSTEM B

TIMES AND CHARGES (CØNT.)
****

		CØNNECT		CPU		I/Ø		TØTAL
TASK	RUN	(HRS)	($)	(SEC)	($)	(SEC)	($)	CHARGE
7	1	.219	2.19	77.5	3.87	34.6	1.04	7.10
	2	.217	2.17	77.8	3.89	32.5	0.98	7.03
	3	.297	2.97	79.1	3.95	33.0	0.99	7.91
AVERAGE		.244	2.44	78.1	3.91	33.4	1.00	7.35
8	1	.540	5.40	26.2	1.31	115.4	3.46	10.17
	2	.271	2.71	25.4	1.27	104.3	3.13	7.11
	3	.214	2.14	25.1	1.26	102.7	3.08	6.47
AVERAGE		.342	3.42	25.6	1.28	107.5	3.22	7.92
9	1	.251	2.51	19.8	0.99	22.8	0.68	4.19
	2	.170	1.70	20.1	1.01	22.0	0.66	3.36
	3	.192	1.92	19.5	0.97	21.5	0.64	3.54
AVERAGE		.204	2.04	19.8	0.99	22.1	0.66	3.70
ØVERALL AVERAGE		.129	1.29	20.3	1.01	26.2	0.78	3.09

SYSTEM C

TIMES AND CHARGES
****

		CONNECT		CPU		I/Ø		TØTAL
TASK	RUN	(HRS)	($)	(SEC)	($)	(SEC)	($)	CHARGE
1	1	.012	0.14	8.5	1.27			1.41
	2	.008	0.09	8.5	1.27			1.36
	3	.013	0.14	8.5	1.27			1.42
AVERAGE		.011	0.12	8.5	1.27	-----	-----	1.40
2	1	.051	0.56	8.5	1.27			1.84
	2	.056	0.61	8.5	1.27			1.89
	3	.056	0.62	9.5	1.42			2.05
AVERAGE		.054	0.60	8.8	1.32	-----	-----	1.92
3	1	.050	0.55	0.5	0.07			0.62
	2	.050	0.55	0.5	0.07			0.62
	3	.049	0.54	0.5	0.07			0.62
AVERAGE		.050	0.55	0.5	0.07	-----	-----	0.62
4	1	.097	1.06	0.5	0.07			1.14
	2	.110	1.21	0.5	0.07			1.28
	3	.089	0.97	0.5	0.07			1.05
AVERAGE		.098	1.08	0.5	0.07	-----	-----	1.16
5	1	.049	0.54	0.5	0.07			0.62
	2	.059	0.65	0.5	0.07			0.73
	3	.036	0.39	0.5	0.07			0.47
AVERAGE		.048	0.53	0.5	0.07	-----	-----	0.60
6	1	.022	0.24	13.5	2.02			2.27
	2	.019	0.20	12.5	1.87			2.08
	3	.017	0.19	12.5	1.87			2.06
AVERAGE		.019	0.21	12.8	1.92	-----	-----	2.14

SYSTEM C

TIMES AND CHARGES (CONT.)
****

		CONNECT		CPU		I/O		TOTAL
TASK	RUN	(HRS)	($)	(SEC)	($)	(SEC)	($)	CHARGE
7	1	.204	2.25	70.5	10.57			12.82
	2	.211	2.32	72.5	10.87			13.20
	3	.215	2.36	71.5	10.72			13.09
AVERAGE		.210	2.31	71.5	10.72	-----	-----	13.04
8	1	.165	1.81	9.5	1.42			3.24
	2	.151	1.66	8.5	1.27			2.93
	3	.155	1.70	8.5	1.27			2.98
AVERAGE		.157	1.72	8.8	1.32	-----	-----	3.05
9	1	.180	1.98	4.5	0.67			2.66
	2	.181	1.99	5.5	0.82			2.82
	3	.181	1.99	5.5	0.82			2.81
AVERAGE		.181	1.99	5.2	0.77	-----	-----	2.76
OVERALL AVERAGE		.092	1.01	13.0	1.95			2.97

SYSTEM D

TIMES AND CHARGES
****

TASK	RUN	CONNECT (HRS)	CONNECT ($)	CPU (SEC)	CPU ($)	I/O (SEC)	I/O ($)	TOTAL CHARGE
1	1	.027	0.27	8.5	1.27			1.54
	2	.026	0.26	8.5	1.27			1.53
	3	.044	0.44	8.8	1.32	-----	-----	1.76
AVERAGE		.032	0.32	8.6	1.29			1.61
2	1	.055	0.55	10.1	1.51			2.06
	2	.056	0.56	10.4	1.56			2.12
	3	.055	0.55	9.7	1.45	-----	-----	2.00
AVERAGE		.055	0.55	10.1	1.51			2.06
3	1	.051	0.51	4.2	0.63			1.14
	2	.054	0.54	4.2	0.63			1.17
	3	.057	0.57	4.0	0.60	-----	-----	1.17
AVERAGE		.054	0.54	4.1	0.62			1.16
4	1	.087	0.87	6.5	0.97			1.84
	2	.112	1.12	6.2	0.93			2.05
	3	.089	0.89	6.3	0.94	-----	-----	1.84
AVERAGE		.096	0.96	6.3	0.95			1.91
5	1	.046	0.46	3.7	0.55			1.01
	2	.035	0.35	3.7	0.55			0.90
	3	.050	0.50	3.7	0.55	-----	-----	1.06
AVERAGE		.044	0.44	3.7	0.55			0.99
6	1	.021	0.21	21.9	3.28			3.50
	2	.021	0.21	22.0	3.30			3.51
	3	.122	1.22	22.0	3.30	-----	-----	4.52
AVERAGE		.055	0.55	22.0	3.29			3.84

SYSTEM D

TIMES AND CHARGES (CØNT.)
****

TASK	RUN	CØNNECT (HRS)	CØNNECT ($)	CPU (SEC)	CPU ($)	I/Ø (SEC)	I/Ø ($)	TØTAL CHARGE
7	1	.199	1.99	55.6	8.34			10.33
	2	.227	2.27	55.9	8.38			10.65
	3	.253	2.53	55.3	8.29			10.83
AVERAGE		.226	2.26	55.6	8.34	-----	-----	10.60
8	1	.261	2.61	25.8	3.87			6.48
	2	.176	1.76	26.4	3.96			5.72
	3	.274	2.74	27.1	4.06			6.80
AVERAGE		.237	2.37	26.4	3.96	-----	-----	6.34
9	1	.216	2.16	21.4	3.21			5.37
	2	.215	2.15	21.1	3.16			5.32
	3	.187	1.87	21.6	3.24			5.11
AVERAGE		.206	2.06	21.4	3.20	-----	-----	5.27
ØVERALL AVERAGE		.112	1.12	17.6	2.64			3.75

SYSTEM E

TIMES AND CHARGES

TASK	RUN	CONNECT (HRS)	CONNECT ($)	CPU (SEC)	CPU ($)	I/O (SEC)	I/O ($)	TOTAL CHARGE
1	1	.036	0.36	11.0	1.65			2.01
	2	.039	0.39	11.3	1.69			2.09
	3	.033	0.33	11.4	1.71			2.04
AVERAGE		.036	0.36	11.2	1.68	-----	-----	2.05
2	1	.086	0.86	14.1	2.11			2.97
	2	.067	0.67	13.9	2.08			2.75
	3	.090	0.90	12.9	1.93			2.84
AVERAGE		.081	0.81	13.6	2.04	-----	-----	2.85
3	1	.049	0.49	5.6	0.84			1.33
	2	.047	0.47	5.4	0.81			1.28
	3	.056	0.56	5.6	0.84			1.40
AVERAGE		.051	0.51	5.5	0.83	-----	-----	1.34
4	1	.137	1.37	8.2	1.23			2.60
	2	.208	2.08	8.0	1.20			3.28
	3	.170	1.70	7.7	1.15			2.86
AVERAGE		.172	1.72	8.0	1.19	-----	-----	2.91
5	1	.047	0.47	4.1	0.61			1.09
	2	.041	0.41	4.1	0.61			1.03
	3	.039	0.39	4.1	0.61			1.00
AVERAGE		.042	0.42	4.1	0.61	-----	-----	1.04
6	1	.044	0.44	34.0	5.10			5.54
	2	.100	1.00	34.6	5.19			6.19
	3	.091	0.91	34.0	5.10			6.01
AVERAGE		.078	0.78	34.2	5.13	-----	-----	5.91

SYSTEM E

TIMES AND CHARGES (CØNT.)
****

TASK	RUN	CØNNECT (HRS)	CØNNECT ($)	CPU (SEC)	CPU ($)	I/Ø (SEC)	I/Ø ($)	TØTAL CHARGE
7	1	.355	3.55	98.8	14.82			18.37
	2	.401	4.01	98.3	14.74			18.76
	3	.520	5.20	97.9	14.68			19.89
AVERAGE		.426	4.26	98.3	14.75	-----	-----	19.01
8	1	.197	1.97	30.5	4.57			6.55
	2	.223	2.23	29.9	4.48			6.72
	3	.211	2.11	30.4	4.56			6.67
AVERAGE		.210	2.10	30.3	4.54	-----	-----	6.64
9	1	.212	2.12	28.0	4.20			6.32
	2	.262	2.62	28.5	4.27			6.89
	3	.276	2.76	28.3	4.24			7.01
AVERAGE		.250	2.50	28.3	4.24	-----	-----	6.74
ØVERALL AVERAGE		.150	1.50	25.9	3.89			5.39

SYSTEM F

TIMES AND CHARGES

****

TASK	RUN	CONNECT (HRS)	CONNECT ($)	CPU (SEC)	CPU ($)	I/Ø (SEC)	I/Ø ($)	TØTAL CHARGE
1	1	.010	0.11	8.5	1.27			1.39
	2	.021	0.23	8.5	1.27			1.51
	3	.033	0.36	8.5	1.27			1.64
AVERAGE		.021	0.24	8.5	1.27	-----	-----	1.51
2	1	.065	0.71	8.5	1.27			1.99
	2	.052	0.57	8.5	1.27			1.85
	3	.057	0.63	8.5	1.27			1.90
AVERAGE		.058	0.64	8.5	1.27	-----	-----	1.91
3	1	.049	0.54	1.5	0.22			0.77
	2	.056	0.61	1.5	0.22			0.84
	3	.054	0.59	1.5	0.22			0.81
AVERAGE		.053	0.58	1.5	0.22	-----	-----	0.81
4	1	.102	1.12	1.5	0.22			1.35
	2	.125	1.38	1.5	0.22			1.60
	3	.105	1.15	1.5	0.22			1.38
AVERAGE		.111	1.22	1.5	0.22	-----	-----	1.44
5	1	.067	0.74	1.5	0.22			0.96
	2	.055	0.61	1.5	0.22			0.83
	3	.049	0.53	1.5	0.22			0.76
AVERAGE		.057	0.63	1.5	0.22	-----	-----	0.85
6	1	.026	0.28	20.5	3.07			3.36
	2	.049	0.54	20.5	3.07			3.62
	3	.024	0.27	20.5	3.07			3.34
AVERAGE		.033	0.36	20.5	3.07	-----	-----	3.44

SYSTEM F

TIMES AND CHARGES (CØNT.)
****

TASK	RUN	CØNNECT (HRS)	CØNNECT ($)	CPU (SEC)	CPU ($)	I/Ø (SEC)	I/Ø ($)	TØTAL CHARGE
7	1	.266	2.93	144.5	21.67			24.60
	2	.201	2.21	135.5	20.32			22.54
	3	.237	2.61	137.5	20.62			23.23
AVERAGE		.235	2.58	139.2	20.87	-----	-----	23.46
8	1	.293	3.23	27.5	4.12			7.35
	2	.221	2.43	25.5	3.82			6.26
	3	.214	2.35	26.5	3.97			6.33
AVERAGE		.243	2.67	26.5	3.97	-----	-----	6.65
9	1	.195	2.14	33.5	5.02			7.17
	2	.197	2.17	32.5	4.87			7.04
	3	.191	2.10	32.5	4.87			6.97
AVERAGE		.194	2.14	32.8	4.92	-----	-----	7.06
ØVERALL AVERAGE		.112	1.23	26.7	4.01			5.24

SYSTEM G

TIMES AND CHARGES

		CØNNECT		CPU		I/Ø		TØTAL
TASK	RUN	(HRS)	($)	(SEC)	($)	(SEC)	($)	CHARGE
1	1	.006	0.06	9.5	0.19			0.25
	2	.007	0.07	10.5	0.21			0.28
	3	.006	0.06	9.5	0.19			0.25
AVERAGE		.007	0.07	9.8	0.20	-----	-----	0.26
2	1	.046	0.46	10.5	0.21			0.67
	2	.046	0.46	9.5	0.19			0.65
	3	.046	0.46	9.5	0.19			0.65
AVERAGE		.046	0.46	9.8	0.20	-----	-----	0.65
3	1	.045	0.45	1.5	0.03			0.48
	2	.045	0.45	1.5	0.03			0.48
	3	.047	0.47	1.5	0.03			0.50
AVERAGE		.046	0.46	1.5	0.03	-----	-----	0.49
4	1	.058	0.58	1.5	0.03			0.61
	2	.059	0.59	1.5	0.03			0.62
	3	.057	0.57	0.5	0.01			0.58
AVERAGE		.058	0.58	1.2	0.02	-----	-----	0.61
5	1	.021	0.21	0.5	0.01			0.22
	2	.019	0.19	0.5	0.01			0.20
	3	.018	0.18	1.5	0.03			0.21
AVERAGE		.019	0.19	0.8	0.02	-----	-----	0.21
6	1	.012	0.12	20.5	0.41			0.53
	2	.012	0.12	21.5	0.43			0.55
	3	.012	0.12	21.5	0.43			0.55
AVERAGE		.012	0.12	21.2	0.42	-----	-----	0.55

SYSTEM G

TIMES AND CHARGES (CØNT.)

		CØNNECT		CPU		I/Ø		TØTAL
TASK	RUN	(HRS)	($)	(SEC)	($)	(SEC)	($)	CHARGE
7	1	.170	1.70	750.5	15.01			16.71
	2	.173	1.73	754.5	15.09			16.82
	3	.194	1.94	852.5	17.05			18.99
		----	-----	-----	-----	-----	-----	-----
AVERAGE		.179	1.79	785.8	15.72			17.51
8	1	.126	1.26	155.5	3.11			4.37
	2	.123	1.23	150.5	3.01			4.24
	3	.114	1.14	149.5	2.99			4.13
		----	-----	-----	-----	-----	-----	-----
AVERAGE		.121	1.21	151.8	3.04			4.25
9	1	.134	1.34	38.5	0.77			2.11
	2	.133	1.33	37.5	0.75			2.08
	3	.133	1.33	38.5	0.77			2.10
		----	-----	-----	-----	-----	-----	-----
AVERAGE		.133	1.33	38.2	0.76			2.10
ØVERALL AVERAGE		.069	0.69	113.4	2.27			2.96

SYSTEM H

TIMES AND CHARGES

		CONNECT		CPU		I/O		TOTAL
TASK	RUN	(HRS)	($)	(SEC)	($)	(SEC)	($)	CHARGE
1	1	.010	0.10	10.5	0.21			0.31
	2	.015	0.15	9.5	0.19			0.34
	3	.011	0.11	10.5	0.21			0.32
AVERAGE		.012	0.12	10.2	0.20			0.32
2	1	.055	0.55	12.5	0.25			0.80
	2	.049	0.49	12.5	0.25			0.74
	3	.052	0.52	12.5	0.25			0.77
AVERAGE		.052	0.52	12.5	0.25			0.77
3	1	.049	0.49	8.5	0.17			0.66
	2	.051	0.51	9.5	0.19			0.70
	3	.049	0.49	9.5	0.19			0.68
AVERAGE		.050	0.50	9.2	0.18			0.68
4	1	.058	0.58	9.5	0.19			0.77
	2	.058	0.58	10.5	0.21			0.79
	3	.061	0.61	9.5	0.19			0.80
AVERAGE		.059	0.59	9.8	0.20			0.79
5	1	.021	0.21	8.5	0.17			0.38
	2	.019	0.19	7.5	0.15			0.34
	3	.017	0.17	7.5	0.15			0.32
AVERAGE		.019	0.19	7.8	0.16			0.35
6	1	.027	0.27	28.5	0.57			0.84
	2	.020	0.20	23.5	0.47			0.67
	3	.016	0.16	23.5	0.47			0.63
AVERAGE		.021	0.21	25.2	0.50			0.72

SYSTEM H

TIMES AND CHARGES (CONT.)
****

TASK	RUN	CONNECT (HRS)	CONNECT ($)	CPU (SEC)	CPU ($)	I/O (SEC)	I/O ($)	TOTAL CHARGE
7	1	.154	1.54	150.5	3.01			4.55
	2	.163	1.63	167.5	3.35			4.98
	3	.160	1.60	156.5	3.13			4.73
AVERAGE		.159	1.59	158.2	3.16	-----	-----	4.75
8	1	.127	1.27	70.5	1.41			2.68
	2	.136	1.36	72.5	1.45			2.81
	3	.141	1.41	69.5	1.39			2.80
AVERAGE		.135	1.35	70.8	1.42	-----	-----	2.76
9	1	.187	1.87	120.5	2.41			4.28
	2	.161	1.61	120.5	2.41			4.02
	3	.168	1.68	122.5	2.45			4.13
AVERAGE		.172	1.72	121.2	2.42	-----	-----	4.14
OVERALL AVERAGE		.075	0.75	47.2	0.94			1.70

SYSTEM I

TIMES AND CHARGES
****

		CØNNECT		CPU		I/Ø		TØTAL
TASK	RUN	(HRS)	($)	(SEC)	($)	(SEC)	($)	CHARGE
1	1	.009	0.10	3.6	1.19			1.29
	2	.009	0.09	3.6	1.18			1.27
	3	.006	0.07	3.6	1.18	-----	-----	1.25
AVERAGE		.008	0.09	3.6	1.18			1.27
2	1	.048	0.53	3.7	1.21			1.74
	2	.044	0.49	3.7	1.21			1.70
	3	.044	0.49	3.7	1.21	-----	-----	1.69
AVERAGE		.046	0.50	3.7	1.21			1.71
3	1	.044	0.48	0.2	0.06			0.54
	2	.043	0.48	0.2	0.06			0.54
	3	.044	0.48	0.2	0.06	-----	-----	0.54
AVERAGE		.044	0.48	0.2	0.06			0.54
4	1	.072	0.79	0.3	0.09			0.88
	2	.072	0.79	0.3	0.09			0.88
	3	.073	0.80	0.3	0.09	-----	-----	0.89
AVERAGE		.072	0.80	0.3	0.09			0.89
5	1	.037	0.41	0.2	0.06			0.47
	2	.036	0.39	0.2	0.05			0.45
	3	.034	0.37	0.2	0.05	-----	-----	0.43
AVERAGE		.036	0.39	0.2	0.05			0.45
6	1	.013	0.14	7.1	2.36			2.50
	2	.013	0.14	7.2	2.37			2.51
	3	.013	0.15	7.1	2.36	-----	-----	2.50
AVERAGE		.013	0.14	7.2	2.36			2.50

SYSTEM I

TIMES AND CHARGES (CØNT.)
****

		CØNNECT		CPU		I/Ø		TØTAL
TASK	RUN	(HRS)	($)	(SEC)	($)	(SEC)	($)	CHARGE
7	1	.167	1.84	45.9	15.15			16.99
	2	.162	1.78	46.0	15.19			16.97
	3	.181	1.99	46.0	15.19			17.18
AVERAGE		.170	1.87	46.0	15.18	-----	-----	17.05
8	1	.144	1.58	14.1	4.64			6.22
	2	.135	1.48	14.1	4.64			6.12
	3	.138	1.52	14.1	4.64			6.16
AVERAGE		.139	1.53	14.1	4.64	-----	-----	6.16
9	1	.152	1.67	2.8	0.93			2.61
	2	.146	1.61	2.8	0.93			2.54
	3	.149	1.64	2.8	0.93			2.57
AVERAGE		.149	1.64	2.8	0.93	-----	-----	2.57
ØVERALL AVERAGE		.075	0.83	8.7	2.86			3.68

SYSTEM J

TIMES AND CHARGES

TASK	RUN	CONNECT (HRS)	CONNECT ($)	CPU (SEC)	CPU ($)	I/O (SEC)	I/O ($)	TOTAL CHARGE
1	1	.008	0.09	1.8	0.60			0.69
	2	.007	0.07	1.8	0.60			0.67
	3	.010	0.11	1.8	0.61			0.71
AVERAGE		.008	0.09	1.8	0.60	-----	-----	0.69
2	1	.050	0.55	2.1	0.70			1.25
	2	.051	0.56	2.1	0.70			1.26
	3	.050	0.55	2.1	0.70			1.25
AVERAGE		.050	0.55	2.1	0.70	-----	-----	1.25
3	1	.049	0.54	0.2	0.07			0.61
	2	.049	0.53	0.2	0.08			0.61
	3	.049	0.53	0.2	0.08			0.61
AVERAGE		.049	0.54	0.2	0.07	-----	-----	0.61
4	1	.078	0.86	0.3	0.11			0.96
	2	.068	0.75	0.3	0.11			0.85
	3	.075	0.83	0.3	0.11			0.94
AVERAGE		.074	0.81	0.3	0.11	-----	-----	0.92
5	1	.027	0.30	0.2	0.07			0.37
	2	.029	0.32	0.2	0.07			0.39
	3	.034	0.38	0.2	0.07			0.45
AVERAGE		.030	0.33	0.2	0.07	-----	-----	0.40
6	1	.015	0.17	4.5	1.47			1.64
	2	.015	0.16	4.5	1.48			1.65
	3	.014	0.16	4.5	1.48			1.64
AVERAGE		.015	0.16	4.5	1.48	-----	-----	1.64

SYSTEM J

TIMES AND CHARGES (CØNT.)
****

		CØNNECT		CPU		I/Ø		TØTAL
TASK	RUN	(HRS)	($)	(SEC)	($)	(SEC)	($)	CHARGE
7	1	.475	5.23	47.5	15.66			20.89
	2	.215	2.37	47.3	15.60			17.97
	3	.317	3.48	47.3	15.62			19.10
AVERAGE		.336	3.69	47.4	15.63	-----	-----	19.32
8	1	.156	1.72	8.4	2.76			4.48
	2	.256	2.82	8.4	2.77			5.59
	3	.146	1.61	8.4	2.76			4.37
AVERAGE		.186	2.05	8.4	2.76	-----	-----	4.81
9	1	.148	1.63	5.3	1.76			3.39
	2	.154	1.69	5.3	1.76			3.45
	3	.151	1.66	5.3	1.75			3.41
AVERAGE		.151	1.66	5.3	1.76	-----	-----	3.42
ØVERALL AVERAGE		.100	1.10	7.8	2.58			3.67

SYSTEM K

TIMES AND CHARGES

		CONNECT		CPU		I/O		TOTAL
TASK	RUN	(HRS)	($)	(SEC)	($)	(SEC)	($)	CHARGE
1	1	.009	0.07	15.1	0.75			0.83
	2	.010	0.08	15.4	0.77			0.85
	3	.010	0.07	15.0	0.75			0.82
AVERAGE		.010	0.07	15.2	0.76	-----	-----	0.83
2	1	.054	0.40	16.5	0.82			1.23
	2	.055	0.41	16.7	0.83			1.25
	3	.056	0.42	16.9	0.84			1.26
AVERAGE		.055	0.41	16.7	0.83	-----	-----	1.25
3	1	.050	0.37	2.0	0.10			0.47
	2	.050	0.37	2.6	0.13			0.50
	3	.050	0.37	2.5	0.13			0.50
AVERAGE		.050	0.37	2.4	0.12	-----	-----	0.49
4	1	.077	0.58	6.0	0.30			0.88
	2	.076	0.57	5.1	0.25			0.83
	3	.130	0.98	6.2	0.31			1.29
AVERAGE		.094	0.71	5.8	0.29	-----	-----	1.00
5	1	.112	0.84	2.5	0.13			0.96
	2	.025	0.19	3.2	0.16			0.35
	3	.025	0.19	1.3	0.06			0.25
AVERAGE		.054	0.40	2.3	0.12	-----	-----	0.52
6	1	.018	0.14	18.8	0.94			1.08
	2	.017	0.13	18.6	0.93			1.06
	3	.018	0.14	18.6	0.93			1.07
AVERAGE		.018	0.13	18.7	0.93	-----	-----	1.07

SYSTEM K

TIMES AND CHARGES (CØNT.)

TASK	RUN	CØNNECT (HRS)	CØNNECT ($)	CPU (SEC)	CPU ($)	I/Ø (SEC)	I/Ø ($)	TØTAL CHARGE
7	1	.402	3.02	315.3	15.77			18.78
	2	.299	2.24	314.5	15.72			17.96
	3	.330	2.47	315.9	15.79			18.27
AVERAGE		.344	2.58	315.2	15.76	-----	-----	18.34
8	1	.165	1.24	78.1	3.90			5.14
	2	.152	1.14	73.3	3.66			4.81
	3	.165	1.24	75.1	3.76			4.99
AVERAGE		.161	1.20	75.5	3.77	-----	-----	4.98
9	1	.157	1.17	17.6	0.88			2.05
	2	.172	1.29	17.2	0.86			2.15
	3	.169	1.27	17.1	0.86			2.12
AVERAGE		.166	1.24	17.3	0.86	-----	-----	2.11
ØVERALL AVERAGE		.106	0.79	52.1	2.61			3.40

SYSTEM L

TIMES AND CHARGES

TASK	RUN	CONNECT (HRS)	CONNECT ($)	CPU (SEC)	CPU ($)	I/Ø (SEC)	I/Ø ($)	TØTAL CHARGE
1	1	.011	0.09	12.3	0.62			0.70
	2	.011	0.08	12.3	0.62			0.70
	3	.009	0.07	11.8	0.59			0.66
AVERAGE		.010	0.08	12.1	0.61	-----	-----	0.69
2	1	.079	0.60	13.5	0.67			1.27
	2	.053	0.40	14.2	0.71			1.11
	3	.054	0.40	14.2	0.71			1.11
AVERAGE		.062	0.47	14.0	0.70	-----	-----	1.16
3	1	.051	0.38	2.3	0.12			0.49
	2	.051	0.38	2.4	0.12			0.50
	3	.051	0.38	2.8	0.14			0.52
AVERAGE		.051	0.38	2.5	0.12	-----	-----	0.51
4	1	.150	1.12	7.3	0.37			1.49
	2	.091	0.68	7.2	0.36			1.04
	3	.132	0.99	7.2	0.36			1.35
AVERAGE		.124	0.93	7.2	0.36	-----	-----	1.29
5	1	.074	0.55	5.6	0.28			0.83
	2	.037	0.28	5.2	0.26			0.54
	3	.050	0.37	4.9	0.24			0.62
AVERAGE		.054	0.40	5.2	0.26	-----	-----	0.66
6	1	.030	0.22	17.9	0.89			1.12
	2	.030	0.23	16.9	0.84			1.07
	3	.019	0.14	18.0	0.90			1.04
AVERAGE		.026	0.20	17.6	0.88	-----	-----	1.08

SYSTEM L

TIMES AND CHARGES (CØNT.)
****

		CØNNECT		CPU		I/Ø		TØTAL
TASK	RUN	(HRS)	($)	(SEC)	($)	(SEC)	($)	CHARGE
7	1	.185	1.39	70.9	3.54			4.93
	2	.208	1.56	69.9	3.49			5.05
	3	.188	1.41	71.4	3.57			4.98
AVERAGE		.194	1.45	70.7	3.54	-----	-----	4.99
8	1	.294	2.21	26.6	1.33			3.54
	2	.191	1.43	25.3	1.26			2.70
	3	.173	1.30	24.8	1.24			2.54
AVERAGE		.220	1.65	25.6	1.28	-----	-----	2.92
9	1	.161	1.21	17.9	0.89			2.11
	2	.158	1.19	17.4	0.87			2.06
	3	.159	1.19	17.2	0.86			2.05
AVERAGE		.159	1.20	17.5	0.87	-----	-----	2.07
ØVERALL AVERAGE		.100	0.75	19.2	0.96			1.71

STØRAGE CHARGES
******* *******

	SYSTEM					
TASK	A	B	C	D	E	F
1	.10	.10	.19	.24	.24	.19
2	.10	.10	.19	.24	.24	.19
3	.10	.10	.19	.24	.24	.19
4	.10	.10	.19	.24	.24	.19
5	.10	.10	.19	.24	.24	.19
6	.10	.10	.19	.24	.48	.19
7	.70	1.00	.56	2.40	2.40	.56
8	.80	1.00	.38	2.16	2.40	.56
9	1.30	1.60	.75	4.08	3.84	.94
AVE.	.38	.47	.31	1.12	1.15	.35

	SYSTEM					
TASK	G	H	I	J	K	L
1	.06	.13	.10	.10	.06	.09
2	.06	.13	.10	.10	.06	.09
3	.06	.06	.10	.10	.06	.06
4	.06	.13	.10	.10	.06	.09
5	.06	.06	.10	.10	.06	.06
6	.06	.06	.10	.10	.06	.06
7	.83	1.02	.60	.80	.72	.95
8	.83	1.09	.60	.90	.69	1.01
9	1.47	1.79	1.10	1.40	1.30	1.67
AVE.	.39	.50	.32	.41	.34	.45

LABØR CØSTS
***** *****

SYSTEM

TASK	A	B	C	D	E	F
1	.05	.04	.03	.18	.21	.10
2	.08	.11	.06	.07	.25	.09
3	.01	.03	.04	.07	.05	.07
4	.45	.49	.28	.26	.79	.37
5	.25	.53	.20	.17	.16	.27
6	.08	.08	.05	.30	.46	.14
7	3.49	.60	.36	.47	1.87	.53
8	1.37	1.54	.25	.81	.63	.85
9	.15	.50	.33	.51	.82	.43
AVE.	.66	.44	.18	.32	.58	.32

SYSTEM

TASK	G	H	I	J	K	L
1	0.00	.04	.01	.01	.02	.03
2	.00	.04	0.00	.03	.06	.12
3	.02	.04	0.00	.04	.05	.05
4	0.00	.00	.10	.11	.25	.46
5	.00	0.00	.12	.08	.24	.24
6	0.00	.06	.00	.02	.04	.10
7	.14	0.00	.08	1.24	1.29	.24
8	0.00	.10	.12	.46	.28	.69
9	0.00	.27	.11	.12	.23	.18
AVE.	.02	.06	.06	.23	.27	.23

TØTAL CØSTS
****

BASIC SYSTEMS

TASK	RUN	A	C	D	G	I	K
1	1	1.16	1.63	1.96	0.32	1.39	0.91
	2	1.16	1.58	1.95	0.35	1.38	0.93
	3	1.31	1.64	2.18	0.32	1.35	0.91
2	1	1.94	2.09	2.37	0.73	1.84	1.35
	2	1.93	2.14	2.43	0.71	1.80	1.37
	3	1.78	2.29	2.31	0.71	1.79	1.39
3	1	0.79	0.86	1.45	0.56	0.64	0.58
	2	0.83	0.85	1.48	0.56	0.64	0.61
	3	0.75	0.85	1.49	0.59	0.64	0.60
4	1	2.95	1.61	2.35	0.68	1.08	1.19
	2	3.33	1.75	2.55	0.69	1.08	1.14
	3	3.26	1.52	2.34	0.65	1.09	1.60
5	1	2.43	1.01	1.42	0.29	0.68	1.26
	2	1.13	1.12	1.32	0.27	0.66	0.65
	3	1.07	0.86	1.47	0.28	0.64	0.55
6	1	1.52	2.50	4.03	0.60	2.60	1.17
	2	1.56	2.31	4.05	0.62	2.61	1.15
	3	1.48	2.30	5.06	0.62	2.61	1.16
7	1	33.17	13.74	13.20	17.69	17.67	20.80
	2	30.46	14.12	13.53	17.80	17.64	19.98
	3	25.44	14.01	13.70	19.96	17.86	20.28
8	1	12.67	3.87	9.45	5.20	6.94	6.11
	2	13.44	3.56	8.70	5.07	6.84	5.78
	3	12.75	3.60	9.78	4.96	6.88	5.96
9	1	4.55	3.74	9.96	3.59	3.82	3.58
	2	4.45	3.90	9.91	3.55	3.75	3.68
	3	4.49	3.90	9.69	3.57	3.78	3.65
AVERAGE		6.36	3.46	5.19	3.37	4.06	4.01

TØTAL CØSTS
****

FØRTRAN SYSTEMS

TASK	RUN	B	E	F	H	J	L
1	1	0.99	2.45	1.68	0.48	0.80	0.81
	2	0.99	2.54	1.80	0.51	0.78	0.81
	3	0.94	2.48	1.93	0.49	0.83	0.77
2	1	1.79	3.46	2.26	0.97	1.38	1.47
	2	1.65	3.24	2.12	0.91	1.39	1.31
	3	1.87	3.32	2.18	0.94	1.38	1.32
3	1	0.83	1.62	1.02	0.76	0.75	0.60
	2	0.92	1.57	1.09	0.81	0.75	0.61
	3	0.83	1.68	1.07	0.79	0.75	0.63
4	1	2.73	3.63	1.90	0.90	1.17	2.04
	2	4.00	4.32	2.16	0.92	1.06	1.59
	3	2.47	3.89	1.93	0.93	1.15	1.90
5	1	3.59	1.49	1.41	0.44	0.55	1.13
	2	2.21	1.43	1.29	0.41	0.57	0.84
	3	2.11	1.41	1.21	0.39	0.63	0.92
6	1	1.38	6.48	3.69	0.97	1.76	1.27
	2	1.46	7.13	3.95	0.79	1.77	1.23
	3	1.35	6.94	3.67	0.76	1.76	1.19
7	1	8.70	22.64	25.69	5.58	22.93	6.13
	2	8.63	23.02	23.63	6.00	20.00	6.25
	3	9.51	24.15	24.33	5.75	21.14	6.18
8	1	12.72	9.57	8.77	3.86	5.84	5.24
	2	9.65	9.74	7.67	3.99	6.94	4.40
	3	9.02	9.69	7.74	3.98	5.72	4.24
9	1	6.29	10.98	8.53	6.34	4.91	3.96
	2	5.46	11.55	8.41	6.08	4.98	3.91
	3	5.64	11.67	8.34	6.19	4.94	3.90
AVERAGE		3.99	7.12	5.91	2.26	4.32	2.39

TØTAL CØSTS
****

VENDØRS

TASK	RUN	U	V	W	X	Y	Z
1	1	1.07	1.66	2.21	0.40	1.10	0.86
	2	1.08	1.69	2.24	0.43	1.08	0.87
	3	1.13	1.78	2.33	0.40	1.09	0.84
2	1	1.86	2.17	2.91	0.85	1.61	1.41
	2	1.79	2.13	2.83	0.81	1.59	1.34
	3	1.82	2.24	2.82	0.83	1.59	1.35
3	1	0.81	0.94	1.54	0.66	0.69	0.59
	2	0.87	0.97	1.53	0.68	0.69	0.61
	3	0.79	0.96	1.59	0.69	0.69	0.62
4	1	2.84	1.75	2.99	0.79	1.13	1.61
	2	3.67	1.95	3.43	0.80	1.07	1.36
	3	2.87	1.73	3.12	0.79	1.12	1.75
5	1	3.01	1.21	1.46	0.37	0.61	1.20
	2	1.67	1.20	1.37	0.34	0.62	0.74
	3	1.59	1.03	1.44	0.33	0.64	0.74
6	1	1.45	3.10	5.26	0.78	2.18	1.22
	2	1.51	3.13	5.59	0.71	2.19	1.19
	3	1.42	2.98	6.00	0.69	2.18	1.18
7	1	20.93	19.72	17.92	11.63	20.30	13.46
	2	19.54	18.87	18.28	11.90	18.82	13.11
	3	17.47	19.17	18.93	12.86	19.50	13.23
8	1	12.69	6.32	9.51	4.53	6.39	5.67
	2	11.54	5.61	9.22	4.53	6.89	5.09
	3	10.88	5.67	9.74	4.47	6.30	5.10
9	1	5.42	6.13	10.47	4.96	4.37	3.77
	2	4.96	6.15	10.73	4.82	4.36	3.79
	3	5.06	6.12	10.68	4.88	4.36	3.77
AVERAGE		5.18	4.68	6.15	2.81	4.19	3.20

BIBLIOGRAPHY

GENERAL ECONOMIC LITERATURE

Adams, Walter, ed. The Structure of American Industry. 3rd ed. New York: Macmillan, 1961.

Backman, Jules. Advertising and Competition. New York: New York University, 1967.

Bain, Joe S. Barriers to New Competition. Cambridge: Harvard University Press, 1956.

________. Industrial Organization. 2nd ed. New York: Wiley, 1968.

Blaug, M. Economic Theory in Retrospect. Homewood, Ill.: Irwin, 1962.

Boulding, Kenneth E. Economic Analysis: Volume I - Microeconomics. 4th ed. New York: Harper and Row, 1966.

Brennan, Michael J. Theory of Economic Statics. Englewood Cliffs, N. J.: Prentice-Hall, 1965.

Burns, Arthur Robert. The Decline of Competition. New York: McGraw-Hill, 1936.

Caves, Richard. American Industry: Structure, Conduct, Performance. 2nd ed. Englewood Cliffs, N. J.: Prentice-Hall, 1967.

Chamberlin, Edward Hastings. The Theory of Monopolistic Competition. Cambridge: Harvard University Press, 1933.

Due, John F. and Clower, Robert W. Intermediate Economic Analysis. 5th ed. Homewood, Ill.: Irwin, 1966.

Edwards, Corwin D. Big Business and the Policy of Competition. Cleveland, Ohio: Western Reserve, 1956.

Fellner, William. Competition Among the Few. New York: Knopf, 1949.

Ferguson, C. E. Microeconomic Theory. Homewood, Ill.: Irwin, 1966.

Galbraith, John Kenneth. The New Industrial State. Boston: Houghton Mifflin, 1967.

Henderson, James Mitchell, and Quandt, Richard E. Microeconomic Theory: A Mathematical Approach. New York: McGraw-Hill, 1958.

Leftwich, Richard H. The Price System and Resource Allocation. 3rd ed. New York: Holt, Rinehart and Winston, 1966.

Naylor, Thomas H. and Vernon, John M. Microeconomics and Decision Models of the Firm. New York: Harcourt, Brace & World, 1969.

Nemmers, Erwin Esser. Managerial Economics: Text and Cases. New York: Wiley, 1966.

Newman, Philip Charles. The Development of Economic Thought. Englewood Cliffs, N. J.: Prentice Hall, 1952.

________; Gayer, Arthur D.; and Spencer, Milton H., ed. Source Readings in Economic Thought. New York: Norton, 1954.

Pegrum, Dudley F. Public Regulation of Business. Revised ed. Homewood, Ill.: Irwin, 1965.

Robinson, Joan. The Economics of Imperfect Competition. London: Macmillan, 1933.

Samuelson, Paul A. Economics. 8th ed. New York: McGraw-Hill, 1970.

Stigler, George J. The Organization of Industry. Homewood, Ill.: Irwin, 1968.

________. The Theory of Price. Revised ed. New York: Macmillan, 1952.

Watson, Donald Stevenson, ed. Price Theory in Action. Boston: Houghton Mifflin, 1965.

Weiss, Leonard W. Case Studies in American Industry. New York: Wiley, 1967.

TIME-SHARING AND THE TIME-SHARING INDUSTRY

Bauer, Walter F., and Hill, Richard H. "Economics of Time-Shared Computing Systems." DATAMATION, November, 1967, pp. 48-55, and December, 1967, pp. 41-49.

Briggs, Peter L. "Future Time-Sharing Technology." COMPUTERWORLD, March 25, 1970, p. S10.

Bueschel, Richard T.; Stephenson, Andrew G.; and Whitney, Douglas C. Commercial Time-Sharing and Utilities. American Management Association, 1969.

Chow, Gregory C. "Technological Change and the Demand for Computers." The American Economic Review, December, 1967, pp. 1117-30.

Cordtz, Dan. "The Coming Shake-Up in Telecommunications." FORTUNE, April, 1970, p. 69.

Fano, R. M., and Corbato, F. S. "Time-Sharing on Computers." Scientific American, September, 1966, pp. 129-40.

Farina, Mario V. Programming in BASIC: The Time-Sharing Language. Englewood Cliffs, N. J.: Prentice-Hall, 1968.

"General Purpose Time-Sharing Survey." COMPUTERWORLD, March 25, 1970, pp. S6-S8.

"GE Steps on Teletype's Toes." Business Week, April 5, 1969, p. 52.

Glauthier, T. James. "Computer Time-Sharing: Its Origins and Development." COMPUTERS and AUTOMATION, October, 1967, p. 24.

Gold, Michael M. "Time-Sharing and Batch Processing: An Experimental Comparison of Their Values in a Problem-Solving Situation." Communications of the ACM, May, 1969, pp. 249-59.

Gruenberger, Fred, ed. The Transition to On-Line Computing. Washington: Thompson, 1967.

Guise, Robert F. Jr. "The '69 Time-Sharing Gold Rush." DATAMATION, August, 1969, p. 38.

Hammersmith, Alan G. "Selecting a Vendor of Time-Shared Computer Services." COMPUTERS and AUTOMATION, October, 1968, p. 21.

Hootman, Joseph T. "The Pricing Dilema." DATAMATION, August, 1969, pp. 61-66.

"How the High-Fliers Take Off." Business Week, November 22, 1969, pp. 112-16.

Huggins, Phyllis. "Guise Reviews T/S User's Buying Habits." COMPUTERWORLD, March 25, 1970, p. 49.

Irwin, Manley R. "Small Business and Time-Shared Computer Systems." The Quarterly Review of Economics and Business, Spring, 1967, pp. 21-29.

________. "The Computer Utility: Market Entry in Search of Public Policy." The Journal of Industrial Economics, July, 1969, pp. 239-52.

Kanter, H.; Moore, A.; and Singer, N. "The Allocation of Computer Time by University Computer Centers." The Journal of Business of the University of Chicago, July, 1968, pp. 375-84.

Karplus, Walter J., ed. On-Line Computing. New York: McGraw-Hill, 1967.

Lundell, Drake. "GE's Information Systems De-Emphasizes Time-Sharing Services for Small User." COMPUTERWORLD, April 22, 1970, p. 1.

________. "Why the Shakeout?" COMPUTERWORLD, March 18, 1970, p. 54.

Main, Jeremy. "Computer Time-Sharing--Everyman at the Console." FORTUNE, August, 1967, p. 91.

Martin, James. Design of Real-Time Computer Systems. Englewood Cliffs, N. J.: Prentice-Hall, 1967.

________. Programming Real-Time Computer Systems. Englewood Cliffs, N. J.: Prentice-Hall, 1965.

Meador, Larry. "Computer Time-Sharing." Chemical Engineering, January 13, 1969, pp. 109-14.

Orr, William D., ed. Conversational Computers. New York: Wiley, 1968.

Parkhill, D. F. The Challenge of the Computer Utility. Reading, Mass.: Addison-Wesley, 1966.

Popell, Steven D.; Burns, David F.; Johnson, Richard C.; Kilcoyne, J. Gerard; Lehmann, Jean-Pierre; Veasy, Eugene B.; Velissaropoulos, Pandelis; Vitry, Goeffroy De; Weitzenkorn, Lee R.; and Widdoes, Thomas G. Computer Time-Sharing. Englewood Cliffs, N. J.: Prentice-Hall, 1966.

Rosenberg, Arthur M. "The Brave New World of Time-Sharing Operating Systems." DATAMATION, August, 1969, p. 47.

Scherr, Allan Lee. An Analysis of Time-Shared Computer Systems. Cambridge: M.I.T. Press, 1967.

Schwab, Bernhard. "The Economics of Sharing Computers." Harvard Business Review, September-October, 1968, pp. 61-70.

Sharpe, William F. The Economics of Computers. New York: Columbia University Press, 1969.

Stimler, Saul. "Some Criteria for Time-Sharing System Performance." Communications of the ACM, January, 1969, pp. 47-53.

Szuprowicz, Bohdan O. "The Time-Sharing Users: Who Are They?" DATAMATION, August, 1969, pp. 55-56.

"The Round AT&T Lost." Business Week, September 6, 1969, p. 68.

Ziegler, James R. Time-Sharing Data Processing Systems. Englewood Cliffs, N. J.: Prentice-Hall, 1967.

EXPERIMENTAL DESIGN AND STATISTICAL ANALYSIS

Bowker, Albert H., and Lieberman, Gerald J. Engineering Statistics. Englewood Cliffs, N. J.: Prentice-Hall, 1959.

Bradley, James V. Distribution-Free Statistical Tests. Englewood Cliffs, N. J.: Prentice-Hall, 1968.

Chou, Ya-lun. Statistical Analysis. New York: Holt, Rinehart, and Winston, 1969.

Duncan, Acheson J. Industrial Statistics and Quality Control. 3rd ed. Homewood, Ill.: Irwin, 1965.

Hicks, Charles R. Fundamental Concepts in the Design of Experiments. New York: Holt, Rinehart and Winston, 1964.

Li, C. C. Introduction to Experimental Statistics. New York: McGraw-Hill, 1964.

Noether, Gottfried E. Elements of Nonparametric Statistics. New York: Wiley, 1967.

Ostle, Bernard. Statistics in Research. 2nd ed. Ames, Iowa: Iowa State University, 1963.

Siegel, Sidney. Nonparametric Statistics for the Behavioral Sciences. New York: McGraw-Hill, 1956.

Walsh, John E. Handbook of Nonparametric Statistics, III. Princeton, N. J.: Van Nostrand, 1968.